EXTERNAL IMBALANCES AND CAPITAL FLOWS

by

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Chapter 1

Introduction

This doctoral thesis focuses on international flows of human and financial capital. It addresses two main topics: i) financial capital flows, current account and exchange rate dynamics in industrial countries; ii) human capital flows from poor regions to developed countries and optimal immigration policies.

Current account deficits and surpluses have reached nowadays record levels in some major economic areas. Given the extent of the phenomenon, many policy makers have questioned the sustainability of current trends and the implications for the adjustment of external positions. Some observers fear that the adjustment process may require large exchange rate swings with disruptive implications for global economic activity. Others argue that the current degree of financial integration should facilitate the absorption of global imbalances.[1]

In light of these considerations, the first part of this thesis analyzes current account dynamics and the developments affecting the accumulation of external debt. In particular, we study the dynamics that characterize industrial countries. Indeed, as we will show in the following, large amounts of external liabilities are nowadays a peculiar feature of the industrial world. This work focuses on small and large open economies in the industrial world; having said that, we also remark the features that are common with adjustment episodes in emerging countries.

In US, both the existence and the persistence of current account deficits have been a matter of fact since the 1980s. Given the historic peak of the dollar and of the current account

[1] For some discussion, see among many others IMF (2005a), Krugman (2007).

deficit during the 1980s, Krugman (1985, 1988, 1989) used a modified version of the Mundell-Fleming (MMF) model to assess the sustainability of the dollar in light of "feasible"[2] levels of US external debt. His back-of-the-envelope calculations showed that the dollar should have inevitably declined – as it eventually happened.

In our work, we have updated Krugman's workhorse model so as to track the accumulation of external debt in a small open economy.[3] This simplified framework has provided insights into balance of payments adjustment processes in presence of significant amounts of foreign currency assets. In this scenario – as it is nowadays the case for most industrial countries – currency depreciation implies significant shifts of wealth from the rest of the world towards the domestic country. Therefore, contrary to what happens for developing countries – which have been generally characterized by significant amounts of foreign denominated debt and suffer therefore from balance sheets effects –, currency depreciation facilitates the adjustment of current account deficits both through the improvement of the terms of trade and assets revaluation effects. The study also analyzes adjustment dynamics when international assets are not perfect substitutes.

The MMF model is an useful workhorse for policy issues in international macroeconomics. However, it is not microfounded. In order to improve on the caveats[4] associated to this, we have extended our analysis and built a microfounded framework. Moreover, we have incorporated some peculiar features that have characterized recent trends in international finance. In light of recent developments in the housing market, we have focused on the spillovers deriving from the housing market and their impact on balance of payments dynamics. The analysis is set in a DSGE New Keynesian framework so as to account for market imperfections and price rigidities. While far from being complete, this framework allows analyzing the international transmission of stochastic shocks that are simulated numerically by using standard perturbation methods. Moreover, the model incorporates an analytical characterization of the steady state.

In the recent years, the flows of human capital from emerging and developing countries

[2] According to Krugman (1988, 1989) a currency is not sustainable when external debt grows larger than the economy's ability to repay it; in practice "the exchange rate is unsustainable if it needs to decline faster than the market expects". In turn, the current account is "feasible" if it does not grow explosively while following a rational expectations trajectory.

[3] We provide an application on UK current trends.

[4] In particular, it lacks of a theoretical linkage between international capital and trade flows and an intertemporal approach to the balance of payments.

toward the industrial world have significantly increased. Indeed, as the IMF suggests, as population aging continues, pressures for migration increase.[5] Relevance to current policy debate has driven our attention towards the dynamics of human capital flows from poor countries towards the industrial world. The second part of the thesis focuses thus on immigration and the implications for destination-countries immigration policies.

Immigration policies are nowadays set by destination countries. The impact of these policies is evident to native citizens so that they are object of great political debate. In his pioneer analysis, Benhabib (1996) has showed the implications of letting native citizens decide over immigration policies. His model shows that if immigration policies are implemented through a referendum and natives are heterogeneous in their wealth endowments, preferences are polarized in favour of complete frontier openness or perfect closure; depending on the median voter's capital endowments, one of these policies is eventually implemented.

Given the current debate on optimal selective immigration policies in Europe, we have extended Benhabib (1996) framework so as to account for immigrants' heterogeneity in their level of skills. This work shows that in a world where immigrants are characterized by different levels of skills and the implementation of policies entails some costs, citizens find optimal to vote for an interior solution: the immigration policy consists in letting at least a minimum amount of both skilled and unskilled immigrants enter the country.

However, optimal immigration policies need also to account for the costs associated to the inflow of immigrants. Indeed, one of the most controversial issues of debate in Western countries concerns the possible economic drain of immigrants on destination countries social programs; the arrival of newcomers imposes policy makers to adapt institutions and social programs to the needs of a new society.

In light of these considerations, we have studied optimal immigration and fiscal policies in modern welfare states. In particular, we have assumed that the government provides individuals with a public good, which is subject to phenomena of congestion, associated to large numbers of recipients. The analysis is set in a dynamic general equilibrium framework and provides a complete characterization of local dynamics and of the stability properties of the steady state.

[5] Moreover, according to IMF (2005b), "it is possible that movements of labour from regions with rising working-age populations to those with rising elderly dependency ratios (as predictions suggest) are a possible alternative to capital flows".

It shows that even if the destination-country policy maker maximizes the welfare of native citizens only, it is always optimal to let (at least) some immigrants enter.

This thesis is structured as follows. In the remaining part of this chapter we provide a short literature review on the two main topics of this thesis. We will first focus on the evidence and the existing literature on financial capital flows and external debt dynamics. We will then review the literature and evidence on human capital flows, which is closest to this work. In Chapter 2, we will focus on the current account dynamics of a small open economy by using an updated version of Krugman (1985) framework.[6] In Chapter 3, we will analyze the dynamics of external imbalances in light of the spillovers deriving from the housing market. In Chapter 4, we shift the attention to human capital flows and selective immigration policies.[7] In Chapter 5, the costs of immigration inflows will be incorporated so as to characterize optimal fiscal and immigration policies.[8] Finally, Chapter 6 provides some concluding remarks.

1.1 Current account and exchange rate dynamics in the industrial world

In the last decades, the world economy has experienced the emergence of record high current account deficits. Indeed, larger external current account deficits or surpluses have been facilitated by the increased scope for cross-border trade in financial assets due to a generalized globalization of financial markets. Figure 1.1 shows the evolution of the sum of world gross external assets and liabilities starting from 1970. It shows that from the end of the 1980s, the

[6] This chapter has been published as "UK external imbalances and the sterling: are they on a sustainable path?" (2007), *Open Economies Review*, **18**, 5, 539-557 (with M. Miller).

[7] This Chapter is forthcoming as "Skills, immigration and selective policies" (2008), in *Zagreb International Review of Economics and Business* (with S. Bosi and F. Magris).

[8] This Chapter has been submitted to the *Journal of Public Economic Theory* as "Optimal immigration policy when the public good is rival" in a shorter version (with S. Bosi and H. Jayet)

pace at which external assets and liabilities accumulated has significantly accelerated.

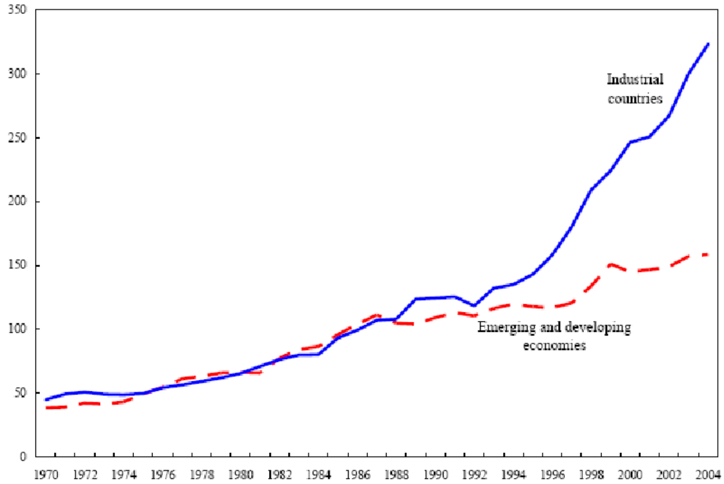


Figure 1.1. Ratio of sum of foreign assets and liabilities to GDP,1970-2004.
Source: Lane and Milesi-Ferretti (2007)

Large international flows of capital have an important precedent during the gold standard period. Indeed, the 50 years before World War I saw massive net private flows of capital from core countries in Western Europe to recent settlements overseas.[9] An interesting difference between the two periods concerns the direction of the flows. At the time, the emerging countries were running current account deficits while the major European economies were surplus countries. Figure 1.2 shows how this trend has reversed throughout the last 30 years; after a peak at the end of the 1990s, emerging countries have changed their saving patterns in favour of a decrease in external liabilities and an accumulation of official reserves.[10]

[9] See IMF (2005a) for an analysis on differences and similarities between international capital flows during the gold-standard period and now.

[10] See also Bernanke (2005), for a discussion on the "saving glut".

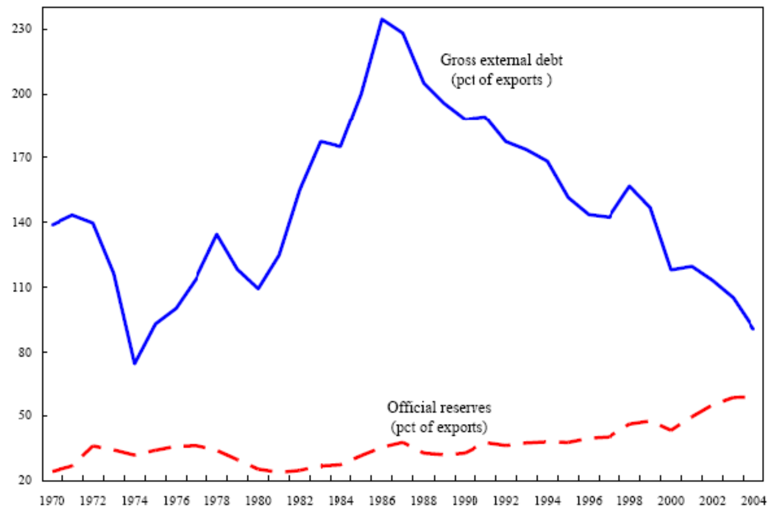


Figure 1.2. External liabilities in the emerging world

Source: Lane and Milesi-Ferretti (2007)

Indeed, during the 1980s and 1990s large amounts of external debt have been often associated to episodes of currency crises[11] and current account reversals in the emerging world.[12] The dramatic effects of the crises have prompted emerging countries to accumulate large amounts of official reserves and international assets for stabilization purposes.[13]

Having said that, past current account reversals and currency crises in the Western world warn that industrialized countries are as well exposed to the risks associated to balance of payments adjustment processes.[14] Therefore, even if higher trade openness and greater competition

[11] Notice however that, as remarked by Milesi-Ferretti and Razin (1998), current account reversals do not necessarily coincide with currency crises. According to Frenkel and Rose (1996), an economy is hit by a currency crash when its currency experiences a nominal depreciation of at least 25% and an increase in the rate of depreciation of at least 10% with respect to the previous year.

[12] See Milesi-Ferretti and Razin (1998) for an empirical analysis on current account reversals and currency crisis in emerging and developing countries during the period 1970-1996. For some theoretical analysis on speculative attacks and balance of payments reversals due to fundamentals see, among others, Flood and Garber (1984) "first generation" model. For some theoretical analysis on currency crisis, external debt and multiple equilibria arising from agents' expectations and coordination failures see, among others, the "second generation" model of speculative attacks of Sachs et al. (1996). For an analytical focus on external debt, exchange rate dynamics and balance sheets effects see, among others, Aghion et al. (2000). These workhorse frameworks have been applied to emerging countries.

[13] For some discussion on saving patterns and sterilization policies in the emerging world see the debate on the emergence of a Bretton Woods II system – among others, see Dooley et al. (2003).

[14] In the period 1980-1995, Freund (2000) individuates 17 episodes of currency crisis – according to the definition of Frankel and Rose – that hit industrialized countries within 2 years from the beginning of a current account adjustment process.

worldwide should likely promote nowadays smoother current account adjustments, the understanding of global imbalances is still an important challenge for economists and policy makers: larger external positions eventually raise economies' exposure to financial market disturbances, and investors' expectations.

While retaining from evaluating the sustainability of current trends, the first part of this thesis aims to unfold the links between current account dynamics, exchange rates and market fundamentals in industrial countries.

The thesis first focuses on a small open economy that is affected by a positive demand shock. The analysis is based on an updated version of Krugman' modified Mundell-Fleming (MMF) framework aiming to evaluate the sustainability of the strong dollar during the 1980s (Krugman 1985, 1988, 1989). We do not use his framework to evaluate the sustainability of the current levels of a currency but to track the dynamics of the external debt. Indeed, as remarked by Krugman, the MMF model is an useful tool for "tracking the performance of the international monetary system and predict the outcomes of policy" (Krugman, 1995). The MMF model is based on a standard Keynesian framework with a demand side determination of output where money supply follows a standard LM function. Net exports depend on relative output and real exchange rates and there is an arbitrage equation for international real interest rates.

We have updated this framework so as to incorporate three key features that have bulked large in the past decade: i) the use of a Taylor rule instead of a monetary target to guide monetary policy; ii) the presence of significant external borrowing to finance accumulated current account deficits; iii) valuation effects. The small open economy we consider is a "cashless economy" without inflation; money is however the unit of account of the economy and the equilibrium in the money market is ensured by a level of interest rates, which is set according to a Taylor rule.[15] The country we consider does not suffer from the "original sin"[16] and can thus borrow in its own currency. There are important implications for the dynamics of external imbalances arising from the denomination of the debt. Balance sheets effects have proved a fundamental mechanism in the context of financial crisis in emerging countries. In the after-

[15] See Woodford (2003) for some discussion on cashless economies.

[16] The "original sin" refers to the fact that some emerging countries have previously defaulted on debt and/or devaluated the currency. Therefore, investors do not accept to lend capital denominated in the currency of the borrower. See Eichengreen et al. (2003a,b) and Reinhart et al. (2003) for some discussion.

math of the Asian crisis, Krugman (1999) showed that if firms have a substantial share of their debt denominated in foreign currency, a confidence loss of international investors can lead to currency depreciation. In turn, negative balance effects on firms' external debt can more than offset the positive impact of the currency on trade and lead to a collapse of domestic investment and aggregate demand. The elegant analysis of Aghion et al. (2000) provides a supply-driven story with analogous implications.[17]

Following the same logic, balance sheet effects modify the accumulation of net external liabilities also if external debt is denominated in domestic currency and assets are denominated in foreign currency. As emphasized by Obstfeld (2004), Obstfeld and Rogoff (2004), Gourinchas and Rey (2007) currency depreciation entails in this case a revaluation of external assets, and thus, a wealth transfer from surplus to deficit countries, which, everything else being equal, can help reduce the amount of trade adjustment that would otherwise be needed.[18] Clearly, the impact of this mechanism is especially strong in presence of large amounts of foreign currency-denominated external assets. This is generally the case of industrial countries, since they tend to have their foreign assets denominated in foreign currency and liabilities in domestic currency. Indeed, according to IMF (2005a), the contribution of valuation effects to changes in net foreign assets during the 1990s has been large relative to current account balances, especially in small open industrial economies.[19]

In our stylized analysis, the accumulation of net external debt is a positive function of the country's trade deficit and the gap between the real interest rate and the growth rate of the economy; it is a negative function of the valuation effects associated with a depreciation of the exchange rate. We first assume that agents can exchange internationally only perfectly substitutable bonds; assets yields coincide with interest rates and the uncovered interest parity

[17] They eventually show that in presence of a negative productivity shock, "it makes sense to increase interest rates [appreciating the exchange rate] when the proportion of foreign currency debt is sufficiently large". Indeed, as proved by turmoils in the late 1990s, balance sheet effects deriving from exchange rate swings can have a dramatic impact on the external wealth of agents.

[18] The impact of valuation effects on the dynamics of foreign assets are affected also by the nature of the exchange rate depreciation/appreciaiton. If the change in the value of the exchange rate is anticipated, they tend to be transitory: indeed they tend to be reflected in asset yields, which, in turn, offset the valuation effects through their impact on the balance of investment incomes.

[19] Morover, the valuation adjustments associated with the US dollar depreciation during 2002–03 have offset about three-fourths of the cumulative US current account deficit. For further evidence see also Lane and Milesi-Ferretti (2005) and Clarida et al. (2005).

condition for non-arbitrage needs to hold in equilibrium. When the real interest rate is larger than the GDP growth rate, an exogenous rise in the aggregate demand triggers a short run appreciation – followed by a depreciation as external debt accumulates and net wealth deteriorates. But when the domestic growth rate exceeds the world real interest rate, the net effect of increased external borrowing is to appreciate the exchange rate. Notice also that the steady state is characterized by a positive level of net external liabilities. This should not surprise the reader; indeed, Lane and Milesi-Ferretti (2001, 2002) have documented the persistence of positive levels of net external liabilities in the long run.

The results of the analysis are conditional upon the fact that assets are perfectly substitutable. However Blanchard et al. (2005) stress that international assets are not perfect substitutes and agents have a bias in favour of domestic assets.[20] Their portfolio analysis individuates a link between the amount of external debt and currency depreciation; they show that in absence of arbitrage opportunities, higher net external debt in US must be associated with a lower value of the dollar.[21]

Following the same logic, we incorporate thus in the analysis the possibility that more international borrowing in home currency involves paying higher interest rates. This implies in practice modifying the international arbitrage condition such that there is a wedge between interest rates, which is a positive function of net external liabilities.[22] The analysis shows that with sufficiently strong home bias an exogenous demand shock will more likely require a depreciation of the currency, even in presence of a negative differential interest rate-growth rate.

The above analysis carries some caveats associated to the lack of microfoundations. As re-

[20] The first article widely cited for noting a bias in portfolios toward home assets was French and Poterba (1991). See among others Tesar and Werner (1995), Baxter and Jermann (1997) and Bottazzi Pesenti and van Wincopp (1996) for more discussion on the home bias puzzle in international assets.

[21] The reason is that as soon as wealth is transferred from the US to the rest of the world, home bias leads to a decrease in the demand for US assets, and thus, a currency depreciation. In the short run, a shift towards dollar denominated assets temporally pushes up the value of the dollar; however, the associated loss of competitiveness entails a larger trade deficit, and thus, a larger external debt. The long-run value of the dollar is eventually more depreciated.

[22] In the case of an emerging country, this wedge could be interpreted as a risk premium; see Schmitt-Grohe and Uribe (2003) for an analysis on the implications deriving from risk premia on international debt – more discussion will also come in the following. Notice however that while in their analysis the risk-premium function is centered around an exogenous level of external debt, in ours it is simply increasing in debt.

marked Obstfeld and Rogoff (1995), by ignoring the intertemporal budget constraint, Keynesian models of international macroeconomics miss a coherent analysis of current account dynamics: by ignoring the relationship between consumption, investment and saving decisions, they do not link the present with the future. Moreover, they do not provide a full description of how monetary policy affects production decisions.

More caveats can be found. Indeed, as remarked by Krugman (1995), the MMF model misses a link with the theory of international trade. While the latter focuses on production and trade patterns in a many-goods, many-factors world, the MMF is a non-microfounded one-good in each-country model.[23]

In light of the above considerations, we introduce a two-country general equilibrium framework to study current account and exchange rate dynamics in industrial countries. We also incorporate the implications arising from the housing sector so as to focus in particular on the mechanism linking international capital flows, exchange rates and agents' real assets.

In recent years, houses have been increasingly used as a collateral, in several industrial countries. The boom of collateralized debt is associated to the housing finance liberalization process. During the past 30 years, deregulation has introduced competitive pressures from non-banking lenders and eventually allowed households a better access to mortgages credit (see Figure 1.3).[24] Collateral constraints insure creditors that in case of default they can repossess (part of) the asset; their existence has thus facilitated better credit conditions for borrowers. By allowing more agents access to collateralized credit, the share of collateral-constraint agents

[23] Finally, as Krugman (1995) eventually concludes, "The fact is that the MMF analysis seems to be extremely useful – it appears to work in practice much better than it ought to work in the light of trade theory."

[24] For a survey on the developments in housing finance in different countries, see IMF (2008).

has eventually boomed [25].

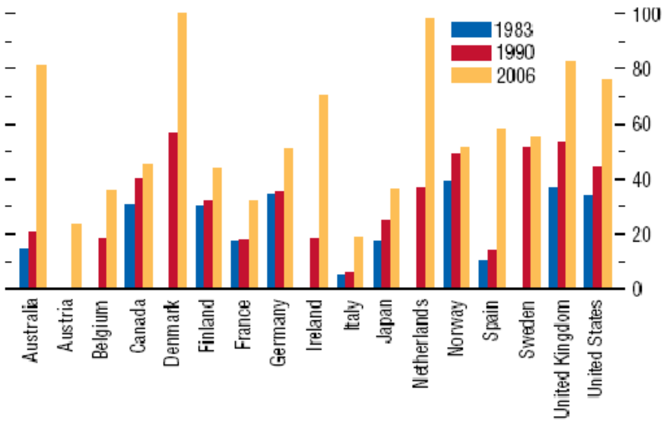


Figure 1.3. Outstanding mortgage debt

Source: IMF (2008)

Developments affecting the housing sector have important implications for consumption patterns. As concluded by IMF (2008) in light of recent empirical evidence, the linkage between house prices and households consumption has strengthened during the last decades. In particular, rising house prices have enhanced consumption via increasing household debt – the "financial accelerator" mechanism (Bernanke et al., 1999).

The role of collateral constraints has been first emphasized by Kiyotaki and Moore (1997) in presence of heterogeneous discounting and durable goods. Their analysis focuses on the transmission of shocks when (relatively more) impatient agents are subject to a collateral debt constraint. As in Bernanke et al. (1999) the "financial accelerator mechanism" amplifies all endogenous developments affecting the credit market and significantly affects the business cycle. Notice however, that when debt contracts are in nominal terms and monetary policy controls interest rates, inflation dynamics amplify demand shocks but dampen supply shocks – working thus as a "financial decelerator" (Iacoviello, 2005). Indeed, with nominal debt contracts, an increase in inflation has a positive effect on debtors' wealth and a negative one of the wealth of lenders; this has thus important implication for optimal monetary policy (see Monacelli, 2007).

[25] In US, mortgage debt has increased from about 60% at the half of the last century to about 90% – including veichles.

Spillovers deriving from housing finance are not only a domestic issue. In the last decades, the globalized structure of financial systems has increasingly allowed financial intermediaries to convert mortgages into assets. Households' debt has been thus sold abroad, and has circulated around the world in the form of international assets. These considerations suggest that the evolution of both international financial markets and house finance has reinforced the link between external assets and real asset prices – and thus, between house prices and international capital flows (see Figure 1.4).

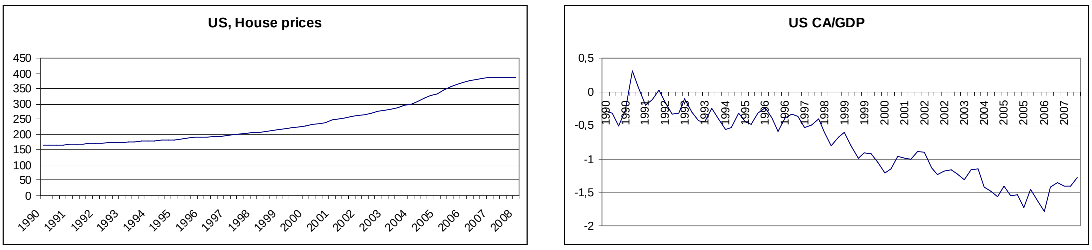


Figure 1.4. Current account and house prices in US.

Source: Ecowin database

In international finance, the role of collateral constraints has been emphasized in the context of currency crises in the emerging world. Indeed, in presence of collateral constraints, the linkage between agents' wealth, exchange rates and the direction of international capital flows has proven to be a very powerful mechanism in amplifying the effects of shocks and investors' expectations swings. In a stylized analysis of the Asian crisis, Krugman (1999) focuses on the link between currency depreciation and the wealth of collateral-constrained firms. He individuates two main mechanisms at the roots of the crisis: i) the above-discussed balance sheet effects and ii) an open-economy "Bernanke-Gertler" effect[26] due to the fact that firms' investments are wealth constrained.[27]

In our framework we aim at extending the open-economy "Bernanke-Gertler" effect to the analysis of current account trends in light of the above-discussed spillovers deriving from the housing market. We aim in particular at analyzing the role played by terms of trade and

[26] See Bernanke and Gertler (1989).

[27] Caballero and Krishnamurthy (2001) build a model on emerging market crises where they analyze the interactions between domestic and international collateral constraints on firms with limited borrowing capacity.

exchange rates in transmitting shocks.

Since we are interested in international amplification-effects related to debt limits, we assume that constrained agents are (relatively more) impatient. The implications of heterogeneous discounting in presence of zero debt-limits have been first analyzed in a closed-economy framework in the seminal articles by Becker (1980) and Becker and Foias (1987). They show that under standard assumptions on preferences and production, the steady-state level of debt coincides with the level implied by the limit. Indeed, debt constraints are a key element to ensure the uniqueness and the dynamic determinacy of the steady state, in presence of agents' heterogeneity in the degree of impatience.[28] We assume in particular that agents in the Home country are more impatient than agents in the Foreign country. In our framework, agents' impatience implies that the collateral constraint of the impatient agent is always binding.

Heterogeneous discounting rates are not new in international finance.[29] Ghironi et al. (2005) introduce heterogeneous discounting in an overlapping generation framework; they show how different degrees of discounting prompt impatient agents to accumulate positive levels of net external liabilities in the long run. At the same time, the absence of intergenerational bequest rules out the patient country to hold the world's wealth in steady state.[30]

More recently, Choi et al. (2008) track the dynamics of US current account by introducing endogenous heterogeneous discounting. By incorporating Uzawa preferences, they assume that agents become more impatient as current consumption increases;[31] the introduction of an endogenous discount rate ensures the dynamic stationarity of the steady state[32] and allows their model to improve the fit of US current account recent trends.

Since we study international spillovers from housing on consumption we assume, as in Mat-

[28] Le Van, Nguyen and Vailakis (2007) show that when agents are heterogeneous in their degree of impatience but there are not debt limits, the steady state consists in an asymtotically-stable optimal trajectory; the steady-state trajectory is characterized by the fact that patient agents hold all the wealth and the impatient agents use all their income to repay debt.

[29] In the context of current account analysis, this hypothesis is also complement of Masson et al. (1994) and Henriksen (2002) who relate current account dynamics with demographic factors. Analogously, Chinn and Prasad (2000) find that demographic factors are significant determinant of the current account balance.

[30] The steady state is endogenously pinned down as a function of the structure of the economy. For more discussion, see also Buiter (1981) and Weil (1989).

[31] The discount rate function is centered around an exogenous level of consumption for the country, assumed to hold in steady state.

[32] See Schmitt-Grohe and Uribe (2003) for an analysis of the dynamic properties associated to endogeneous discounting in a small open economy framework; see Bodenstein (2006) for an analogous analysis in a two country framework.

suyama (1990)[33], that agents' (composite) consumption good is made of durable non-tradable goods and tradable non-durable goods. More precisely, the latter basket is composed by Home and Foreign-produced tradables; the former, by non-tradable real assets.

In our analysis, we extend Monacelli (2007, 2008) and Iacoviello (2005) New-Keynesian closed-economy framework to a two country-world. This framework allows us to analyze the role of both price rigidities and monetary policy in presence of collateral-constrained agents and durable goods. Indeed, while the global dimension of the world economy should help a gradual realignment of current imbalances, price rigidities and market imperfections could play an important role in exchange rate dynamics. As the IMF suggests in this respect, "..given the imperfect global integration of markets for goods and services and the rigidities that constrain the reallocation of resources to tradable sectors, the redistribution of world spending is likely to require considerable movements in real exchange rates.." (IMF, 2007).

Since they provide a microfounded focus on both on current account and real assets prices in the industrialized world, the works that are closest to our analysis are the ones by Callegari (2007) and Punzi (2007). In a busyness cycle framework, Punzi (2007) studies current account and house prices dynamics in presence of heterogeneous agents; similarly, Callegari (2007) focuses on the twin deficit hypothesis[34] by analyzing the impact of different fiscal policies on US trade balance. However, none of these works analyze the above-discussed link of housing trends and international capital flows with exchange rate and relative prices. Indeed, in their business cycle framework there are no exchange rates. Moreover, none of these studies account for the role of monetary policy nor for price rigidities.

Our contribute consists thus in transferring a mechanism, which has proven fundamental in the context of emerging market crises to the analysis of external adjustments. While extending the focus to the developments of housing wealth, we extend in practice the application of the

[33]The work of Matsuyama (1990) is the first one to focus on the linkage between the current account and the housing sector. He analyzes in particular the effects of fiscal policy shocks on the current account. The linkage between current account and durable goods has been also analyzed in an empirical framework; İscan (2002) shows how the introduction of both durables and non-traded goods improves upon a model with traded goods only. Callegari (2007) and Punzi (2007) study current account dynamics and housing in a business cycle framework – see the following discussion.

[34]The twin deficit hypothesis refers to the fact that during the 1980s the US current account deficit was associated to a large budget deficit. The loose fiscal policy has been thus pointed out as being the main cause of the current account deficit.

open-economy "Bernanke-Gertler" effect to the analysis of current account dynamics.

Our work individuates the transmission channel that links housing market developments with external positions; it allows in particular to individuate the linkage between house prices and terms of trade. We will show in the following how this mechanism is at the roots of the international transmission of exogenous shocks. Similarly to Ferrero et al. (2008), the analysis also puts in light the role played by the monetary policy stance.

1.2 Capital flows and immigration policies in destination countries

In recent decades, the flows of human capital from emerging and developing countries toward the industrial world have soared. Indeed, the increasing global dimension of trade and financial transactions has created new challenges for national borders and human capital mobility. Figure 1.5 shows an acceleration of net migration into OECD countries starting from the end of the 1980s. As a reflection of the increased dimension of transactions, labour markets have experienced a significant enlargement.

Demographics is playing a big part in migration too.[35] As suggested by the IMF (2005b), the working age population will about double in 2050; it has quadrupled since 1980 as China and India have joined the world economy with their huge young populations. On the other hand, the rich world experiences declining population growth rates and rising life expectancy. Due to these demographic changes, the EU population in particular is undergoing a long term trend of ageing, leading to a likely fall in the working population in the 25 states from 303 million to 297 million by 2020.[36]

[35] Moreover, while large income differentials likely play the most important role, changes in climate, wars and unexpected geopolitical events could add on current trends and further displace millions of people.

[36] This has important implications for pension systems; for some discussion, see among others Sand and Razin (2007).

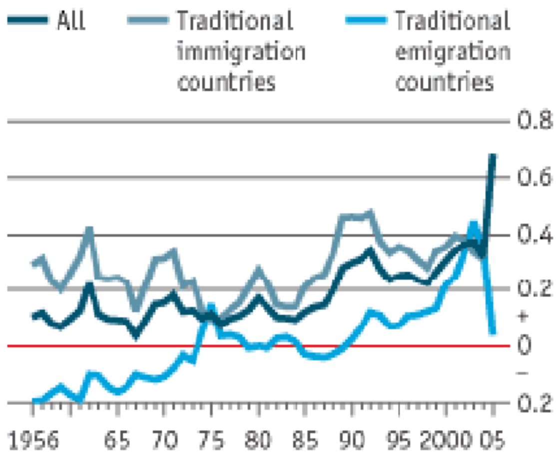


Figure 1.5. Net migration rate into OECD[37]

The above considerations suggest that migration flows can likely represent a boon for the economies of destination countries. In the literature, four types of benefits arising from immigration have been suggested (see Nannestad, 2007): i) the "immigration surplus": the difference between the increase in output and the share of output earned by immigrants; ii) the possible increase in aggregate demand, deriving from the arrival of immigrants; iii) the positive impact on age distribution in the host society; iv) possible positive effects in making the host labor market smoother. While issues concerning social security systems and rigidities in labor markets are nowadays important subjects for policy discussion, they are out of the purpose of this work. In the following we will instead focus on the gains arising from factor complementarity.[38]

Immigrants provide part of the unskilled labour force where it is scarce because of the old aging of the population – as in the industrial world. They can moreover provide specific skills that are both scarce and critical. However, the arrival of immigrants is also associated to significant social costs due to the need of adapting institutions to an heterogeneous population; this could entail also an increase in public expenditure. Indeed, misguided policies or failures to integrate migrants could have dramatic effects on the societies of destination countries.

[37] Net migration rate in OECD countries as a % of resident population. Source: The Economist (2008) from OECD, "Labour Force Statistics 2006"

[38] In Chapter 5, we build a framework where immigrants' impact on aggregate demand is the result of utility maximization programs.

To the purpose of optimizing costs and benefits, immigration flows need to be regulated so as to balance positive and negative effects. In the second part of this thesis, we will focus on the gains and the costs associated to immigration. Our analysis will thus provide a framework for the evaluation and implementation of immigration policies.

Immigration policies and regulations are indeed of great priority in the agenda of international organizations and policy makers in the Western world. Moreover, since the outcomes of immigration regulations are immediately evident to the eyes of native citizens, they are often part of the rhetoric in political campaigns.

Policy relevance in Europe has first driven our attention to study the decision process that leads policy makers to implement immigration regulations in destination countries based on quotas. Since the collapse of the Soviet Union, only few countries restrict their citizen the possibility to migrate (such as Cuba, Myanmar and North-Korea). A part from these few exceptions, migration restrictions are decided by destination countries. We have analyzed the role of heterogeneity in immigrants' skills and the implications associated to the implementation of quotas.

Immigration quotas are commonly used in traditional destination countries of mass migration. In Australia, immigration inflows are regulated by a well articulated legislation. In order to obtain a permanent visa, potential candidates for work visas are subject to an evaluation of their skills: the higher the score, the higher the chances to enter the country. The Canadian immigration system is similar to the Australian one but is articulated in two categories only: skilled work and business immigration. The skills assessment depends on factors such as language skills and work experience. US immigration policies are based on a complicated system of visas, which is articulated in more than 60 temporary visas and some permanent ones. In general, permanent visas are issued only as last step of a long process that starts with a job offer. Obtaining the Green Card is granted only where the labour certification has demonstrated any particular type of skills shortages in US. In the other cases, the process may take several years.

In general, mass immigration flows towards Europe are a relatively recent phenomenon. It is thus not surprising that several European countries still miss a well defined framework for immigration regulations.[39] Having said that, the enlargement of the European Union creates nowadays an additional pressure for coordinate policy-making.

Given the increasing extent of human capital flows, the political debate on immigration policies in Europe has aimed to establish selection criteria with favorable effects on destination-countries' economies. In particular, in countries such as the United Kingdom and France[40], the discussion has been oriented in favour of policies that could possibly discriminate immigrants with respect to their skills (such as the UK Highly Skilled Migrant Program), converging thus to the Australian and Canadian system (and to the logic of the US one).

In light of the above considerations, we build a framework where immigrants are allowed to be heterogeneous in their level of skills. Moreover, when they enter the destination country, they own no capital.[41] To the contrary, we allow natives to be heterogeneous in their degrees of capital. Our aim is to explain the implementation of immigration quotas; interestingly, to our knowledge there have been no systematic research work on this issue.

Bilal et al. (2003) introduce factor complementarity to explain preferences toward specific-skill immigrants. In their framework, only the immigrants who are complementary to the median native in the destination country are welcome. This results are however at odds with reality. Indeed, the above-mentioned immigration systems are characterized by positive quotas both for skilled and unskilled workers.

In a neoclassical framework of growth, Benhabib (1996) introduces heterogeneity in natives' capital to explain different attitudes toward immigration: the stronger the complementarity in endowments between natives and immigrants, the greater natives' incentive to welcome immigrants. However, while immigrants are allowed to be endowed with heterogeneous amounts of capital, they are homogeneous in skills. His results show that, according to capital endowments of the median voter, natives will implement an immigration policy consisting in either completely open either completely closed frontiers.[42]

Our work is built on Benhabib (1996) but is extended to account for labor heterogeneity.

[39] Notice however that the immigration policy is currently a great issue of political debate also in US. Indeed, immigration regulations evolve with the needs of the society.

[40] Notice also that a chronic lack of IT engineers has prompted Germany to facilitate their entry.

[41] Indeed, we aim to model mass immigration flows into rich countries.

[42] In his framework, immigrants can be "selected" only if they have very high or very low endowments of capital. Having said that, there are no quotas for owners of intermediate levesl of capital and all willing-to-be immigrants in that range are wellcome.

Indeed, as remarked also by Benhabib (1996) "An important and interesting one [consideration] has to do with [..] the non-homogeneity of the labor force".

As in his work, we assume that the immigration policy is implemented through a voting process that individuates a *Condorcet* winning policy. Indeed, we need to rule out the implications deriving from Arrow's paradox due to natives' heterogeneity in capital endowments. In a voting procedure à la Condorcet, immigration policies are compared two by two. This mechanism selects the policy that wins against any other alternative, pair by pair, and insures that the larger share of the population is satisfied. The political outcome depends on the capital endowment of the median voter.

We also extend Benhabib (1996) so as to account for the costs associated to the implementation of the immigration regulation. Indeed, the protection of borders can prove very expensive and cannot but be part of the discussion on immigration policies; in US, much of the debate concerning the construction of 12,000 miles of fences in the border between Mexico and US had to deal with the costs associated to it. By the end of 2008, 670 miles of fencing will be eventually built along the 1,969-mile border.[43]

Either (1986) accounts for the impact of enforcement costs on host-countries economies to analyze the role of illegal immigration. In his framework, immigration regulations are a tool to control the unskilled labor rate. However, if illegal immigrants enter the country, such restriction can prove useless and both the wage of unskilled workers and the volume of illegal immigration are distinct policy targets. Border enforcement can prove costly; his analysis eventually shows that it is more convenient for the destination country to employ a mixture of border protection and domestic measures (such as imposing fines to firms that hire illegal immigrants). However, the evidence in US has shown that domestic measures are little effective and need to be enforced as well (implying additional costs).

More recently, Myers and Papageorgiou (2000) account for enforcement costs to focus on fiscal policies. They show that if illegal immigrants can be excluded from the redistribution of public resources, it is optimal for the destination country to close frontiers but spend no resources to enforce it. In this case, illegal immigrants would enter the country but would not

[43] To ensure protection of the Mexican border, US display also gear, gadgets and manpower such as officials in helicopters, jeeps, horseback, on bikes (there are nowadays some 18,000 officiers checking borders and about 15,000 man patrol in-between them – see The Economist (2008)).

weight on the host-country social system.

Finally, Magris and Russo (2005), extend Benhabib (1996) so as to account for enforcement costs. While their framework proves useful to explain interior solution for immigration policies, labour is considered homogeneous; there is thus no room to explain immigration quotas.

We assume that both labor and implementation costs are diversified. Indeed, the implementation costs associated to unskilled workers differ from the ones associated to skilled ones; while the deterrence of unskilled workers is generally enforced through border protection, the screening of skilled workers is mostly associated to bureaucratic costs. In Europe for instance, the commissioner for justice and home affairs is pushing the introduction of a "blue card" to facilitate the entry of highly skilled immigrants; ID cards, databases with biometric details and E-verify systems are planned to tightly police them.[44] On the other hand, the protection of border is insured by Frontex' patrol craft, spotter planes and land-based radar and gears.

In our framework, heterogeneous labor and enforcement costs can indeed explain the introduction of immigration quotas: for a large range of capital endowments, natives vote for positive quotas both for unskilled and skilled immigrants. This result is the outcome of balancing the effects associated to factor complementarity, on the one hand; the ones deriving from enforcement costs associated to quotas, on the other.

The above analysis has proved useful to evaluate costs and benefits that are associated to the introduction of immigration quotas. It is however far from providing a complete understanding of the trade-offs that policy makers face while deciding over immigration policies. Indeed, a part from policy-implementation costs, countries need also to account for the possible drain of immigrants on destination countries' government resources. The issue is particularly tricky as today most Western countries are welfare states[45].

As Nannestad (2007) remarks, there is no general consensus on total fiscal net effects of migration into welfare states. The reason is that net effects critically depend on the assumptions made on: i) immigrants' features; ii) the adopted time perspective; iii) the public good provided by the government and the way s/he raises revenues.[46]

[44] In France, Sarkozy proposed to monitor the immigrants applying for the reunification visa through a DNA test. It is not clear who should eventually carry the costs associated to it.

[45] We simply define the welfare state as a modern state that provides public goods to residents.

[46] Finally, immigration can modify the income distribution of the destination country. Immigration can indeed

In presence of adverse-selection effects due to the generosity of destination-countries' welfare systems, the entrance of immigrants cannot be welfare improving.[47] Having said that, other characteristics can prove critical, such as the age composition of the immigrant population. Clearly, younger workers drain less resources from the social system than older ones. Finally, the extent of immigrants' integration into the hosting labor market plays the most important role: the faster they are integrated, the stronger the contribution to government revenues.[48]

One of the main issues of policy discussion concerns the fact that mass immigrants could likely contribute less than what they cost to welfare states.[49] Given immigrants' lower expected incomes (and a slower integration into the labor market), this may be the case for several European welfare states. Having said that, results can significantly change with the time perspective. Indeed, Smith and Edmonston (1997) estimate that while the short-term net fiscal impact of immigration is negative, the long-term impact is a net positive balance of \$80,000 for each immigrant.[50]

Most of the existing theoretical literature evaluating welfare effects of immigration tends to see the phenomenon as a static one[51]; therefore, these studies do not account for the adjustment dynamics of production factors. In a neoclassical growth framework of overlapping dynasties, Ben-Gad (2004) shows that both the benefits and the fall in wages are much smaller; he argues that static models tend indeed to overestimate the impact of immigration on both the distribution of income and the welfare of natives. The key role of capital is also stressed by Usher (1977) who remarks the importance of a long-term perspective.

Finally, the destination-country fiscal policy itself plays a critical role. In a static framework, Wildasin (1994) shows that when immigrants are beneficiaries of income transfers migration can lead to Pareto-inferior outcomes in the host country. Michael and Hatzipanayotou (2001) show that the effect of immigrants crucially depends on public good provision and on how welfare

have asymmetric income effects. As remarked by Borjas (1990), the immigration surplus is positive if at least some natives see their income decrease.

[47] For some discusison see in Borjas (1999) the "welfare-magnet" hypothesis.

[48] In the following we will assume for simplicity homogenous labor.

[49] The National Research Council argues that in US, each immigrant with lower-than-high school education entail during their life a net loss for the government of \$89,000.

[50] To obtain this estimation, the authors discount the annual fiscal gain/loss over 300 years. Notice however that this result is based on the assumption that the US fiscal policy is currently unsustainable and taxes need to eventually rise.

[51] The common excercise is a static comparison between pre and post-immigration welfare in the country.

policies are financed: in presence of consumption taxes, marginal migration increases natives' welfare in destination countries. Having said that, if revenues of consumption taxes are equally redistributed, marginal migration dampens natives' welfare. More ambiguous outcomes arise from tariffs revenues.

In light of the above considerations, we introduce a dynamic general equilibrium framework to focus on welfare effects of mass immigration. Our aim is to contribute to the current debate on optimal immigration policies in welfare states; we will show in the following that even under relatively conservative assumptions, (regulated) immigration is optimal.

In our framework, the government provides a public good subject to congestion to all residents. Therefore, marginal immigration has a negative effect on the utility that natives derive from its consumption. Moreover, we allow for a degree of free riding in immigrants' contribution[52] to government revenues so that the provision of the public good weights more on natives. Finally, we also incorporate natives' attitudes towards migration. The recent political debate in Europe shows that even if immigration flows have helped sustaining the long-term growth rate in several countries, there is a widespread hostility toward immigration. A poll in 2007 showed that 55% of Spaniards consider immigrants a boon for the economy, and so 50% of Italians, just 42% of Britons and Germans and only 30% of French respondents (The Economist, 2008). In support of this evidence, O'Rourke and Sinnot (2006) and Gang, Rivera-Batiz and Yun (2002) stress that natives' preferences towards immigration are significantly affected by nationalistic sentiments. On the other hand, Bisin and Verdier (2000) and Bisin, Topa and Verdier (2001) remark the significance of immigrants' assimilation resilience. These considerations suggest that the impact of foreign workforce on host countries goes beyond issues related to income distribution; immigration inflows represent a cultural challenge for host countries' natives and, in this light, a disutility[53]

Since we now study the effects of mass migration, we assume that when they enter the destination countries, immigrants own no (physical and human) capital and have homogeneous endowments of labor; nonetheless, they are allowed to accumulate (physical and human) capital

[52] Schultz and Sjöström (2001) propose the accumulation of public debt as a possible tool to mitigate the free-rider problem. Indeed, if the public good is financed by local debt, immigrants will need to pay it by sharing the debt burden. Their two-periods analysis shows that in equilibrium there will be either too much debt and too little public good; this policy recommendation is far from being optimal.

[53] Alternatively, according to Nannestad (2004), integration can be seen as a public good.

as soon as they enter the host country. On the other hand, natives own positive stocks of (human and physical) capital and can decide to further increase their stock of capital; however, capital cannot be neither borrowed neither lent. Moreover, since we don't deal with return migration, we assume that immigrants are more impatient than natives.[54] This assumption has very important implications for the dynamics of the system. As discussed above in an open-economy framework, discounting heterogeneity implies that the patient agent (in this case, the native agent) own all the capital around the steady state; given that the impatient agent (i.e., the immigrant) cannot here access to credit, she/he consumes in each period all her/his labor income.

Our framework allows us to characterize thus the dynamics around the steady state and to focus on the stability properties of our system. In order to study the stability of our system, we use the method of Grandmont et al. (1998); this method is based on a geometrical representation of the trace (T) and determinant (D) of the Jacobian matrix as functions of a convenient bifurcation parameter. We show that, for some parametrizations of our model, two periods cycles can occur. Following Becker and Foias (1994), the bifurcation parameter we consider is the elasticity of labor-capital substitution;[55] indeed, our results show that for sufficiently low degrees of this parameter, a further decrease in the elasticity entails a flip bifurcation and the system passes from saddle-path dynamics to a source. We also focus on the policy options for the social planner to stabilize the economy and on the transition dynamics following shocks in the policy parameters.

Finally, we characterize the optimal immigration and fiscal policy mix. Indeed, since the inflow of immigrants has an impact on government resources and the provision of the public good, the two policies cannot be generally disentangled.[56] Our results show that in absence of costs of cultural heterogeneity, it is optimal to keep frontiers open. However, if natives are

[54]In this respect, from the perspective of immigrants, remittances to source countries can be considered as current expenditure. While remittances account for large share of some source-countries GDP, the study of their impact on source countries is out from the purpose of our study.

[55]Becker and Foias (1994) show that a periodic equilibrium may exist when total capital income is a decreasing function of aggregate capital. In particular, if capital is not too easily substituable, a further decrease in the elasticity of capital-labor substitution entails oscillations in total capital income. Therefore, the income of the patient agent (the capitalist) fluctuates.

[56]Having said that, with specifical functional forms, the number of immigrants does not affect the optimal fiscal policy – see Chapter 5 for all details.

hostile towards immigrants, interior solutions are preferred.

Chapter 2

UK external imbalances and the sterling: are they on a sustainable path?

2.1 Introduction

In his Frank Graham's lecture of 1993 Paul Krugman recommended a MMF or "modified Mundell-Fleming model" as a useful workhorse for policy issues in international macroeconomics[1]. In this essay[2] we update this approach by including three key features which have bulked large in the intervening decade: the use of a Taylor rule instead of a monetary target to guide monetary policy (Taylor, 1993); the presence of significant external borrowing financing accumulated current account deficits[3]; valuation effects[4]. This updated framework, where the Central Bank manipulates real interest rates to stabilize the economy, is used to study the impact of demand shocks on the real exchange rate in the short and long run and to study the strength of the sterling.

[1]See also Krugman (1995).

[2]This chapter has been published as "UK external imbalances and the sterling: are they on a sustainable path?" (2007), *Open Economies Review*, **18**, 5, 539-557 (with M. Miller).

[3]IMF (2005a) discusses how countries have been more prompt to accumulate external liabilities in times of globalization.

[4]See Gourinchas and Rey (2004), Obstfeld and Rogoff (2004), IMF (2005a), Lane and Milesi-Ferretti (2005).

The importance of tracking the course of indebtedness has been emphasized in two papers by Leith, and Wren-Lewis (2000, 2002). In the former, where a Taylor rule is used to raise real interest rates when inflation is above target in a closed economy, their principal conclusion is that fiscal policy needs to be tightened as and when Government debt increases – otherwise rising real interest rates destabilize debt dynamics. In the latter the analysis is carried over to a two open economies model with floating exchange rates. Their elegant analysis involves two special assumptions; that agents have access to perfect capital markets so consumption depends on wealth and not current income; that price setting is as described in Calvo (1983) where the price level is sluggish but not inflation itself.

As with Leith and Wren-Lewis (LWL), we find that the response to a domestic demand shock is a rise in the real interest rates and real exchange rate, which is reversed in the long run– but there has to be a sufficiently strong wealth effect to ensure macroeconomic stability.

We provide an application, namely the UK under current policies. We show how the current imbalances do not necessarily lead to a long term weakening of the currency[5]. Indeed, the strength of the sterling depends crucially on the interaction of UK real growth rate and real interest rates; if UK real interest rates are smaller than its growth rate, long-term equilibrium is consistent with a strong currency. In fact, the accumulation of trade deficits due to the strong currency is offset by the strong growth of the economy. The result is reversed when real interest rates are larger than the growth rate.

Blanchard, Giavazzi and Sa (2005) stress that international liabilities are not necessarily perfect substitutes. In order to take into account for their considerations, we incorporate in our model the effect of home bias. We find that introducing home bias in our model implies smaller imbalances in the long run.

2.2 Stabilizing feedback

Consider a small open economy with no inflation and which does not therefore suffer from Original Sin i.e., it can borrow internationally in its own domestic currency (Eichengreen et al. 2003a and 2003b). Assume that the real interest rate is used to manage aggregate demand,

[5] For further discussion see also Hausmann and Sturzenegger (2006), Meissner and Taylor (2006).

according to a type of Taylor rule; and that the latter depends on the real exchange rate. Then the dynamics of asset accumulation and the real exchange rate can be analyzed in a two-dimensional dynamic system, as follows.

2.2.1 Asset dynamics and the real exchange rate

Consider now a country's net international investment position. Changes in net foreign asset positions are given by net exports of goods and services, current transfers, the investment income balance[6], capital transfers and capital gains. If we ignore current and capital transfers on account of their relatively small size (see also IMF, 2005a) and we assume that real interest rates are equal to real returns, the accumulation of external indebtedness can be represented by the following equation:

$$\dot{d}\left(t\right)=\left(r\left(t\right)-g\left(t\right)\right)\left(d\left(t\right)+\frac{A\left(t\right)}{Q\left(t\right)}\right)+\beta q\left(t\right)-\left(r^{*}\left(t\right)-g\left(t\right)\right)\frac{A\left(t\right)}{Q\left(t\right)}+\frac{A\left(t\right)}{Q\left(t\right)}\dot{q}\left(t\right) \qquad (2.1)$$

where d indicates net external indebtedness (ratio to GDP), q is the log difference between the value of the real exchange rate level that takes trade into balance and the current level (relative price of traded goods – high q means uncompetitive exchange rate), A represents gross foreign currency external assets (ratio to GDP), $\frac{A}{Q}$ indicates gross external assets in domestic currency[7], r is the domestic real interest rate, r^{*} is the rest of the world real interest rate, g is the long run real growth rate and β is the elasticity of trade with respect to the real exchange rate.

Notice in particular that the last term on the right-hand side of equation (2.1) represents the valuation effects stressed by Gourinchas and Rey (2007). For simplicity we have assumed that all gross external assets are denominated in foreign currency and all gross external liabilities are denominated in domestic currency. This hypothesis fits realistically US data: its liabilities are almost entirely denominated in dollars and most of its assets are in foreign currencies. We consider this simplification a reasonable approximation also for other industrial countries'

[6] Where the sum of the three is the current account balance.
[7] Where Q is the real exchange rate level

external positions; industrial countries are indeed generally characterized by a relatively large share of domestic-currency liabilities and large share of foreign-currency assets.

Let us now assume $\frac{A(t)}{Q(t)}$ fixed and define for simplicity $c \equiv \frac{A(t)}{Q(t)}$. Then, assuming r^* and g fixed, equation (2.1) can be rewritten in the following form:

$$\dot{d}(t) = (r(t) - g)d(t) + c(r(t) - r^*) + \beta q(t) + c\dot{q}(t) \tag{2.2}$$

Let now interest rates be used to stabilize output, i.e. a sort of Taylor rule , so,

$$r(t) - r^* = \alpha(y(t) - \bar{y}) \tag{2.3}$$

where y indicates real output (in log) and $\bar{y}$ is the output target for monetary policy.

Let finally the determination of output depend on the following IS curve, i.e.:

$$y(t) - \bar{y} = -\delta q(t) - \gamma r(t) - \eta d(t) + x(t) \tag{2.4}$$

Note that in addition to high interest rates and a high exchange rate, external debt exerts a dampening effect on aggregate demand (proportional to η). In addition we have included the variable x to represent an exogenous component of aggregate demand for domestic goods.

Substituting the Taylor rule (2.3) for real interest rates and aggregate demand (2.4) for real output into equation (2.2), we obtain the following equation for debt accumulation:

$$\dot{d}(t) = \left(\frac{r^* + \alpha x(t)}{1+\alpha\gamma} - g - \theta c\right) d(t) - \theta d^2(t) + q(t)(\beta - \varepsilon d(t) - \varepsilon c) + c\left(\frac{+\alpha(x(t) - r^*\gamma)}{1+\alpha\gamma}\right) + c\dot{q}(t) \tag{2.5}$$

where $\theta \equiv \alpha\eta/(1+\alpha\gamma)$, $\varepsilon \equiv \alpha\delta/(1+\alpha\gamma)$. We also observe that

$$r(t) = \frac{r^* + \alpha x(t) - \alpha\delta q(t) - \alpha\eta d(t)}{1+\alpha\gamma}$$

is UK real interest rate.

2.2.2 International financial arbitrage

The second dynamic equation involves the arbitrage condition. Converting the usual uncovered interest parity condition into real terms implies an arbitrage condition:

$$\dot{q}(t) = r^* - r(t) \tag{2.6}$$

On substituting for r and y, we find:

$$\dot{q}(t) = r^* - r(t) = -\alpha(y(t) - \bar{y}) = -\alpha(-\delta q(t) - \gamma r(t) - \eta d(t) + x(t))$$

So, dropping constant terms and collecting terms in r, we can write

$$\dot{q}(t) = \varepsilon q(t) + \theta d(t) + \frac{\alpha(\gamma r^* - x(t))}{1 + \alpha\gamma} \tag{2.7}$$

2.2.3 External indebtedness and real exchange rate: the dynamics of the system

On combining equation (2.5) with equation (2.7) we now derive the dynamics that link external indebtedness to the evolution of the real exchange rate. On substituting equation (2.7) for valuation effects in equation (2.5), we obtain the following system of differential equations:

$$\begin{aligned} \dot{d}(t) &= \left(\frac{r^* + \alpha x(t)}{1 + \alpha\gamma} - g\right) d(t) - \varepsilon d(t) q(t) - \theta d^2(t) + \beta q(t) \quad (2.8) \\ \dot{q}(t) &= \theta d(t) + \varepsilon q(t) - \left(\frac{\alpha(x(t) - \gamma r^*)}{1 + \alpha\gamma}\right) \quad (2.9) \end{aligned}$$

The stationarity schedules for external indebtedness and the real exchange rate are sketched in Figure 2.1. Notice that at the right of point A on the non-linear locus of stationarity for debt, real interest rates fall below the growth rate.

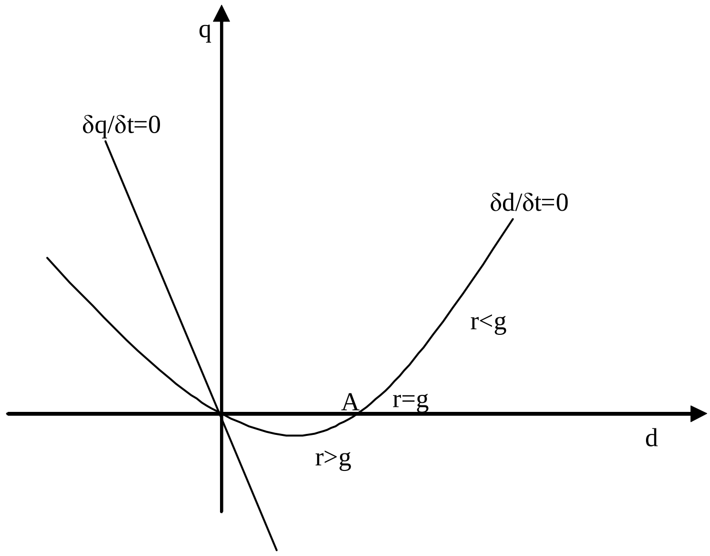


Figure 2.1. Allowing for reversal $r - g$.

Note also that whenever $\varepsilon d(t) - \beta$ approaches zero, q tends to infinity (i.e. external liabilities follow an explosive path; $\varepsilon d(t) - \beta$ represents the asymptote of the curve). We restrict our analysis by assuming $\varepsilon d(t) - \beta < 0$, because for reasonable parameters[8], $\varepsilon d(t) - \beta = 0$ implies net external liabilities are larger than 200% of GDP.

Choosing units so that $\bar{y} = 0$ and and $q(t) = 0$ in equilibrium, and assuming for convenience that $d(t) = 0$, then $\bar{y} = y$ requires that $x(t) = \gamma r^*$. In this case, the constant terms appearing in equation (2.9) disappears and that in equation (2.8) becomes simply $r^* - g$. After linearizing equation (2.9) around equilibrium (in which $q(t) = d(t) = 0$), the dynamic system can then be written as:

$$\begin{bmatrix} \dot{d}(t) \\ \dot{q}(t) \end{bmatrix} = \begin{bmatrix} r^* - g & \beta \\ \theta & \varepsilon \end{bmatrix} \begin{bmatrix} d(t) \\ q(t) \end{bmatrix} \tag{2.10}$$

where the determinant of the Jacobian matrix of (2.10) is $(r^* - g)\varepsilon - \theta\beta$.

While the system will be unstable with no feedback ($\theta = 0$), there will have a saddle point structure as long as $\theta > (r^* - g)\varepsilon/\beta$. Assuming this condition is satisfied, the dynamics are

[8] For parameter estimates we have drawn eclectically on Krugman (1988), Leith and Wren-Lewis (2000, 2002), Obstfeld and Rogoff (2000) and Smith and Wickens (1990).

shown in Figure 2.2 below - where it ensures that the line of stationarity for q is steeper than that for d. Given that in steady state $r^* = r$, the stability condition implies that the extra cost of servicing one unit of external debt, $r - g$, must generate an offsetting improvement in the trade balance in the long term (due to depreciation of the currency induced by the negative wealth effect). Effectively, the negative wealth effect depresses aggregate demand and allows the monetary authority to cut interest rates and so the exchange rate.

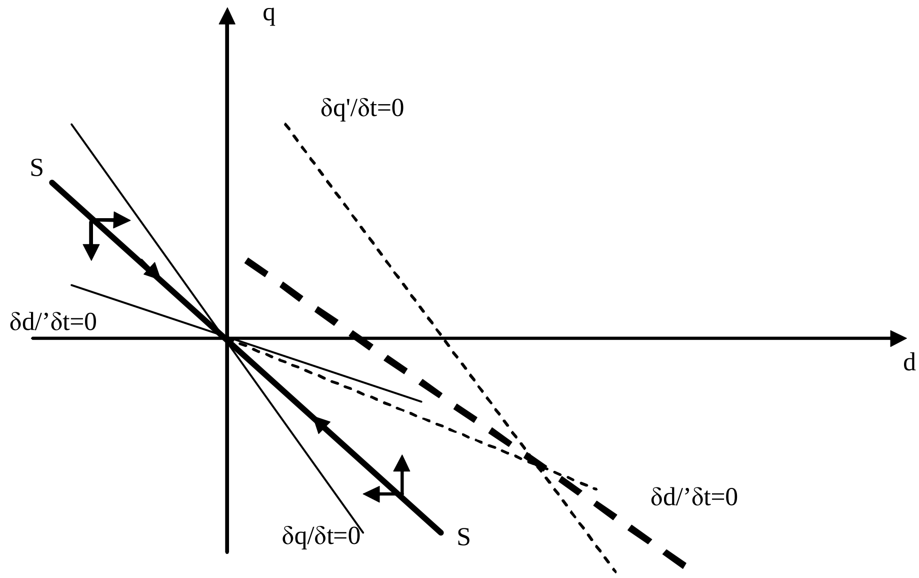


Figure 2.2. Saddlepoint stability when feedback is sufficient

Figure 2.2 is constructed on the assumption that $x = \gamma r^*$ so equilibrium is at the origin. Figure 2.2 also shows the effect of an increase in x both on impact and in the long-run equilibrium. We will discuss in the following the effect of an exogenous demand shock.

2.3 Impact of demand shocks

Let the economy be affected by a positive demand shock and consider for simplicity the effect of an increase in $x(t)$ such that $x(t) + \Delta x(t) \equiv x > \gamma r^*$. It is easy to show that the steady state of the economy is no longer at the origin. The stationarity schedule for external liabilities is in fact characterized by the following equation:

$$q(t) = \frac{\left(\frac{(r^*+\alpha x)}{1+\alpha\gamma} - g\right) d(t) - \theta d^2(t)}{\varepsilon d(t) - \beta} \tag{2.11}$$

where we continue assuming that $\varepsilon d(t) - \beta < 0$.

As a result of the exogenous shock, the curve (2.11) continues to pass through the origin as in Figure 2.2 and 2.3. Note that the minimum of the curve is characterized by a positive level of debt whenever: $\alpha x > g(1+\gamma) - r^*$. We assume this condition to be satisfied (see Appendix, A1). Notice also that the derivative of (2.11) with respect to x is negative; therefore, an exogenous demand shock swivels the curve clockwise (see Figure 2.2-2.3). For what concerns the stationarity schedule for real exchange rate, q, the impact of shocks in the exogenous demand component, implies a rightward shift. Its slope is not affected.

2.3.1 Comparative statics

In order to analyze the long-term effects of an exogenous demand shock, we now focus on the steady-state. Given the system of differential equations (2.8-2.9), steady-state values are given by the following:

$$(d,q)^* = \left(\frac{\beta(x-\gamma r^*)}{\beta\eta + \delta(g-r^*)}, \frac{(x-\gamma r^*)(g-r^*)}{\beta\eta + \delta(g-r^*)}\right)$$

It is possible to show (see (2.12-2.13)) that while an increase in x always determines an increase in the steady-state value for debt, its effect on the steady-state value for real exchange rate is not uniquely signed. In fact, if for example $g - r^* > 0$, an increase in x leads to an appreciation of the currency. If instead $g - r^* < 0$, the real exchange rate needs to depreciate. These considerations imply that, as shown in Figure 2.2 and 2.3, a positive exogenous demand shock always shifts the saddle path rightward.

$$\left(\frac{\partial d^*}{\partial x(t)}\right) = \frac{\beta}{\beta\eta + \delta(g-r^*)} > 0 \tag{2.12}$$

$$\left(\frac{\partial q^*}{\partial x(t)}\right) = \frac{(g-r^*)}{\beta\eta + \delta(g-r^*)} \tag{2.13}$$

Note finally that an increase in η implies a decrease in steady state values for debt and an appreciation of the currency if $g - r^* < 0$, a depreciation if $g - r^* > 0$.

2.3.2 Transition

We now proceed by focusing the attention on the path of the system towards the steady state.

Given the linearized form of the debt schedule:

$$\dot{d} \approx \left(\frac{r^* + \alpha x}{1 + \alpha\gamma} - g - \varepsilon q^* - 2\theta d^* \right) d + q\left(\beta - \varepsilon d^*\right)$$

it is then possible to substitute for the steady-state value of the real exchange rate and write the Jacobian matrix of system (2.8-2.9) in the following form:

$$\begin{bmatrix} r^* - g - \theta d^* & \beta - \varepsilon d^* \\ \theta & \varepsilon \end{bmatrix}$$

As in Section 2, saddle-path stability requires $\beta\theta > \varepsilon(r^* - g)$.

In order to analyze the transition of the system toward steady state, we now focus on the saddle path. Let the general equation for the saddle path (linearized around steady state) be represented by:

$$q\left(t\right) = \frac{v_{21}}{v_{11}} d\left(t\right) + q^* - \frac{v_{21}}{v_{11}} d^* \tag{2.14}$$

where v_{11} and v_{21} are respectively the first and the second component of the stable eigenvector. It is easy to see that the slope of (2.14) is always negative and can be represented by the following:

$$\frac{v_{21}}{v_{11}} = \frac{\theta}{\lambda_s - \varepsilon} = \frac{2\theta}{-\varepsilon + r^* - g - \theta d^* - \sqrt{\left(\varepsilon + r^* - g - \theta d^*\right)^2 + 4\left(\beta\theta - \varepsilon(r^* - g)\right)}} < 0 \tag{2.15}$$

where λ_s is the stable eigenvalue. It is possible to show that an exogenous demand shock (such that $x\left(t\right) > \gamma r^*$) has the effect of swivelling the saddle path anticlockwise; in fact, under very plausible conditions (see Appendix, A2), the derivative of its slope with respect to x is positive.

The above considerations suggest that an exogenous demand shock shifts the saddle-path rightward and turns it anti-clockwise; during the adjustment process, external liabilities accumulate. Whether the real exchange rate depreciates in the long run crucially depends upon $(g-r^*)$. If $g-r^*>0$, the steady state level for the real exchange rate will be more appreciated than before the shock. If $g-r^*<0$, steady state will be characterized by a weaker currency.

2.3.3 Trajectory

As a consequence of an exogenous demand shock (such that $x(t)>\gamma r^*$), the real exchange rate accommodates x in order to keep the economy on a path towards steady state[9]. External liabilities only adjust in the long run. The real exchange rate jumps to a level $q(0)$ – which depends on the predetermined value of $d(t)$ that we indicate as $d(0)$. In the long run, both net external liabilities and the real exchange rate move towards steady state values. It is possible to show that the long term adjustment process from $q(0)$ towards steady state implies a depreciation. Indeed:

$$q(0)-q^*=\frac{\theta}{\lambda_s-\varepsilon}\left(d(0)-d^*\right)>0$$

2.4 Demand shocks, Taylor rules and real exchange rates

An interesting feature of the current situation is that growth rates exceed real interest rates in both US and UK. This is illustrated in the table below where the first line shows yields on indexed debt in the medium and long run. In the US for example these are around 1% in the medium term, slightly less than 2% in the long run i.e. substantially less than current estimates of US growth rate (for which Godley et al. 2004 use a figure of 3.2% for example). For the UK, real interest rates of around one and an half per cent (both medium and long term) are less than the growth of potential GDP, currently estimated at 2.5%. (As a check on these real rates, for the US and the UK we compare them with yields on benchmark bonds to provide the implicit inflation estimates shown in parenthesis in the last row.)

[9] For some discussion on overshooting dynamics see also Buiter and Miller (1982).

	US		UK		Germany	
Real	(2008)	(2028)	(2009)	(2024)	(2009)	(2037)
	1.03	1.93	1.44	1.43	"0.77"	"1.63"
Nominal	(2010)	(2031)	(2010)	(2036)	(2010)	(2037)
	4.25	4.62	4.23	4.27	2.77	3.63
Inflation (estimated)	3.22	2.69	2.79	2.84	"2.0"	"2.0"

Table 2.1. Real and nominal government bond yields, medium and long term.
Source: FT (04/10/05)

As there is no quoted indexed debt on issue in Germany, real rates are crudely estimated by subtracting an inflation forecast from the nominal yields on benchmark government bonds shown in Table 2.1. If German inflation was constant at 2%, for example, this would imply real yields arising from something under 1% in the medium term to about 1.5% in the long term.

2.4.1 Buoyant exogenous demand, monetary policy and external debt

The strength of sterling over the last few years has been widely attributed to buoyant domestic demand and whose inflationary consequences have been checked by tight monetary policy. What are the implications for external indebtedness and exchange rate in the long run?

Starting from the origin let there be a sustained increase in exogenous demand (which can be attributed both to the rise in public expenditure and to the recent real estate bubble), which triggers a rise of real interest rates as the monetary authorities implement a Taylor rule. This has the effect of pushing up the value of the currency so that in the short run it jumps from the equilibrium, 0, to A, the intersection between the vertical axis and the new stable manifold, $S\prime S\prime$ (see Figure 2.3). The currency appreciation and the associated trade deficit lead to a rise in external liabilities; and, given a sufficiently large wealth effect, demand will subside allowing real interest rates to be reduced, i.e. the economy moves down the stable manifold along which external liabilities accumulate and the real exchange rate slowly depreciates.

In equilibrium, the arbitrage condition requires domestic and foreign real interest rates to be equalized, say at 2%. For convenience assume that this is also the growth rate for UK in the long run. The final equilibrium E must then be at a point of trade balance i.e. on the horizontal axis. So the real exchange rate will first rise and then fall to its initial level, despite the increased external indebtedness.

What if UK growth rate lies above the world real interest rate over the long run – as now appears to be the case? In the presence of relatively low real interest rates, countries can sustain a stronger currency than otherwise. Therefore, if present conditions reflect the long term, strong sterling does not imply an explosive growth of UK external imbalances.

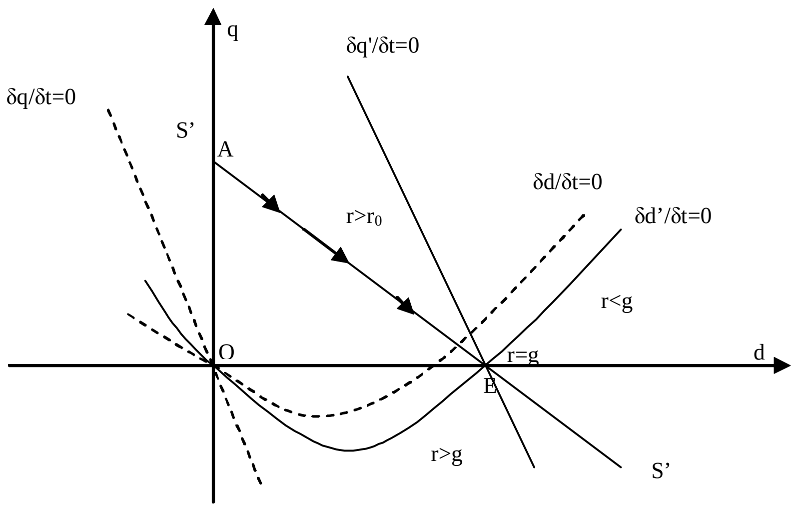


Figure 2.3.UK: an exogenous demand shock in the short and long run

To the extent that buoyant domestic demand has been driven by public sector investment, an interesting issue is whether such an equilibrium will occur with government debt peaking at less than 40% of GDP. If not, then it will trigger the Treasury's "investment sustainability rule" designed to restrain demand directly.

2.5 Effects of an increase in the cost of borrowing

Blanchard et al. (2005) stress home bias, i.e. that more international borrowing in home currency involves paying higher interest rates. Allowing for home bias - in their case issuing more dollars - leads to a fall in the dollar keeping the domestic real interest rate constant. In this application, however, we treat the domestic real rate, r, as endogenous and explore how home bias alters the impact of an exogenous demand shock on indebtedness and competitiveness.

Introducing the hypothesis of imperfect substitutability implies modifying the arbitrage condition such that:

$$\dot{q}(t) = r^* - r(t) + \varphi d(t) \tag{2.16}$$

where φ represents the effect of home bias on investors preferences and is related to the net external indebtedness.

Following the above logic, we now let the economy be affected by a positive exogenous demand shock, such that $x(t) + \Delta x(t) \equiv x > \gamma r^*$. The dynamics of the system are defined by:

$$\dot{d}(t) = \left(\frac{r^* + \alpha x}{1 + \alpha\gamma} - g\right) d(t) - \varepsilon d(t) q(t) - \theta d^2(t) + \beta q(t) \tag{2.17}$$

$$\dot{q}(t) = (\theta + \varphi) d(t) + \varepsilon q(t) - \frac{\alpha(x - \gamma r^*)}{1 + \alpha\gamma} \tag{2.18}$$

By comparing system (2.8-2.9) - i.e. in absence of home bias - with system (2.17-2.18), one difference is evident. Home bias implies a more negatively sloped stationarity schedule for q (sketched in Figure 2.4 with bold lines, both before and after the exogenous demand shock; stationarity schedules in absence of home bias are represented with the dotted lines).

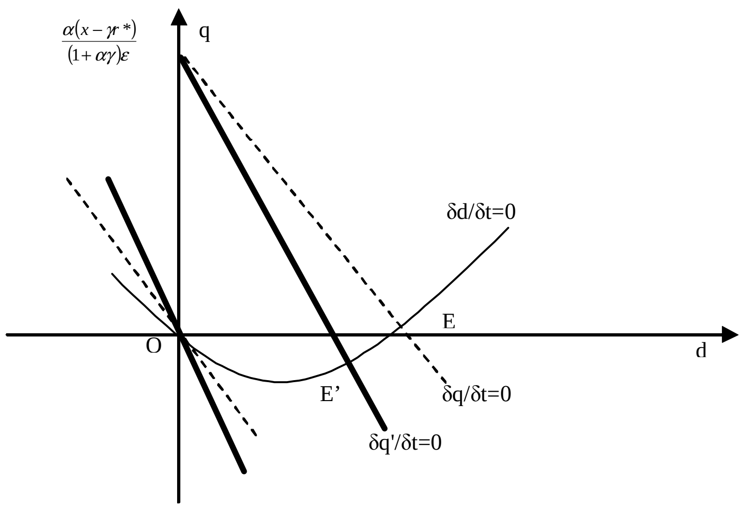


Figure 2.4. Exogenous demand shock with home bias.

2.5.1 Steady state and comparative statics

In order to analyze the long-term effects of an exogenous demand shock in presence of home bias in investors' preferences, we now focus on steady-state values. Given the stationarity schedules for external liabilities and for the real exchange rate (2.17-2.18), the existence of real solutions for steady-state values requires the following condition to hold:

$$\frac{\beta}{\varepsilon}(\theta+\varphi)+g-r^{*}>\sqrt{4\varphi\frac{\beta}{\varepsilon}\frac{\alpha\left(x-\gamma r^{*}\right)}{1+\alpha\gamma}} \tag{2.19}$$

which is verified for φ sufficiently small.

Note that the steady-state value for debt is defined by two roots, d_1^* and d_2^*, suggesting the existence of multiple equilibria; however, one of them can be excluded from the analysis. Under (2.19) d_1^* is real and therefore,

$$0<d_1^*<d_2^*$$

It is possible to show that for $\varepsilon d(t)-\beta<0$, d_1^* is positive and consistent with our analysis, while d_2^* implies $\varepsilon d(t)-\beta>0$. The steady state of the system is defined by:

$$d_1^* = \frac{\frac{\beta}{\varepsilon}(\theta+\varphi)+g-r^{*}-\sqrt{\left(\frac{\beta}{\varepsilon}(\theta+\varphi)+g-r^{*}\right)^{2}-4\varphi\frac{\beta}{\varepsilon}\frac{\alpha(x-\gamma r^{*})}{1+\alpha\gamma}}}{2\varphi} \tag{2.20}$$

$$q_1^* = \frac{\alpha\left(x-\gamma r^{*}\right)}{\varepsilon\left(1+\alpha\gamma\right)}-\frac{\theta+\varphi}{\varepsilon}d_1^* \tag{2.21}$$

An exogenous demand shock leads to an increase in external indebtedness also in presence of home bias (the derivative of (2.20) with respect to x is indeed always positive). As can be seen in Figure 2.4, the shock shifts the steady state from the origin, O, to a point such as $E\prime$ characterized by a positive amount of net external liabilities. In Figure 2.4 we consider an increase in aggregate demand which strengthens or does not weaken the exchange rate in the absence of home bias (see the movement from the origin O to E); in presence of home bias, however, the extent of external debt will be less but the exchange rate will be weaker (see the movement from O to $E\prime$). The basic reason for this is that although there is less debt in equilibrium, the cost of servicing this debt is higher, requiring a larger trade surplus and a lower exchange rate.

If instead we consider an increase in aggregate demand which weakens the exchange rate in the absence of home bias ($r^* > g$), introducing home bias will also lead to a depreciated currency. Note finally that whenever home bias is sufficiently large ($\varphi > \frac{\alpha}{d_1^*}\frac{(x-\gamma r^*)}{(1+\alpha\gamma)} - \theta$), an exogenous demand shock implies a depreciated currency in the long run.

2.5.2 Transition towards steady state

In order to analyze the transition of the economy toward the steady state, we now proceed computing the Jacobian matrix of system (2.17-2.18) (linearized around steady state). Thus:

$$J = \begin{bmatrix} r^* - g - (\theta - \varphi)\, d_1^* & \beta - \varepsilon d_1^* \\ \theta + \varphi & \varepsilon \end{bmatrix}$$

where saddle-path stability requires:

$$\frac{\beta}{\varepsilon}(\theta + \varphi) + g - r^* > 2\varphi d_1^* \tag{2.22}$$

Note however that condition (2.19) implies that (2.22) is always satisfied (see Appendix, A3). Therefore, if steady state exists, then it is a saddle point. The saddle path towards steady state is qualitatively analogous to the saddle path in absence of home bias (compare in fact equation (2.23) and (2.15) letting $\varphi = 0$). Its slope is always negative and it is given by:

$$\frac{v_{21}}{v_{11}} = \frac{\theta + \varphi}{\lambda_s - \varepsilon} = \frac{2(\theta + \varphi)}{r^* - g - (\theta - \varphi)\, d_1^* - \varepsilon - \sqrt{(r^* - g - (\theta - \varphi)\, d_1^* - \varepsilon)^2 + 4(\varphi + \theta)(\beta - d_1^*\varepsilon)}} < 0 \tag{2.23}$$

where v_{11} and v_{21} are respectively the first and the second component of the stable eigenvector and λ_s is the stable eigenvalue. In response to a positive exogenous demand shock the real exchange rate adjusts immediately and jumps to a level $q(0)$.Then, external liabilities accumulate during the transition towards steady state. This adjustment process entails a depreciation of the currency. Indeed:

$$q(0) - q_1^* = \left(\frac{\theta + \varphi}{\lambda_s - \varepsilon}\right)(d(0) - d_1^*) > 0$$

2.5.3 The effects of home bias on the transition process

We now shift the attention towards the impact of different degrees of home bias on the adjustment process. It is possible to show that for reasonable values of the parameters, the introduction of imperfect substitutability implies a more negatively sloped saddle path: the higher the degree of home bias, the more negatively sloped the curve (for some baseline cases, see Table 2.2, 2.3, 2.4 in the Appendix). The basic reason is that the higher cost of servicing the debt implies a stronger currency during the adjustment (see arbitrage equation, (2.16)).(for further discussion on steady-state values see also the appendix A4).

We also provide some baseline cases that show the speed of adjustment towards steady state for different degrees of home bias (see the Figure 2.5 in the Appendix). Note that for φ significant[10], higher degrees of home bias imply a faster adjustment toward steady state. The reason is that the higher cost of servicing debt implied by home bias acts as a constraint for debt accumulation.

2.6 Concluding remarks

We have looked at Taylor rules and debt dynamics in an open economy, emphasizing the role played by the wealth effect in the standard case where the real interest rate is greater than the growth rate. In response to an exogenous rise in the aggregate demand, the dynamics indicate that the short run appreciation followed by a depreciation as external debt accumulates and net wealth deteriorates. But when the domestic growth rate exceeds the world real interest rate, the net effect of increased external borrowing is to raise the exchange rate.

The effect of rising debt on sovereign spreads emphasized by Blanchard et al. can change this result. With sufficiently strong home bias, an exogenous demand will lead to a depreciation

[10]If φ that tends to zero and both x and r^* are relatively large, a marginal increase in home bias makes convergence slower. The reason is that home bias' binding effect on the growth of external liabilities is more than offset by its effect in keeping interest rates high (and therefore the real exchange rate strong) slowing therefore the adjustment process.

of the currency even when the domestic growth rate exceeds the world real interest rate.

A final word of warning: if real interest rates were to return to more normal levels, where the cost of borrowing exceeds the rate of interest for the UK, then extra spending will of course weaken the currency. The UK – and the US – could be in a honeymoon period when more external borrowing is relatively painless; but this honeymoon could end soon.

2.7 Appendix

2.7.1 A1

Consider now the equation defining the stationarity schedule for external liabilities accumulation (see equation 2.11). We now compute its derivative with respect to $d(t)$. We obtain:

$$\left(\frac{\partial q(t)}{\partial d(t)}\right) = \frac{-\theta\varepsilon(1+\gamma\alpha)d^2(t) + 2\beta\theta(1+\gamma\alpha)d(t) + \beta(g(1+\gamma\alpha) - (r^* + \alpha x(t)))}{(1+\gamma\alpha)(\beta - \varepsilon d(t))^2} \tag{2.24}$$

While its denominator is always positive, the sign of 2.24 depends on the numerator. The optima of the function are given by the following (where we exclude the larger root because it implies $\beta - \varepsilon d(t) < 0$ – we restrict the analysis to $\beta - \varepsilon d(t) > 0$):

$$d_{1,2} = \frac{\beta}{\varepsilon} \pm \sqrt{\left(\frac{\beta}{\varepsilon}\right)^2 + \frac{\beta}{\varepsilon}\frac{1}{\theta}(g-z)}$$

where we let $z = (r^* + \alpha x)/(1+\alpha\gamma)$.

The system of equations (2.8-2.9) can present the following features:

1: $z > g + \beta\theta/\varepsilon$, two positive eigenvalues in the dynamic system of equations.

2: $z < g + \beta\theta/\varepsilon$ one positive and one negative eigenvalue in the dynamic system of equations (saddle path).

2.1 $z < g$ one positive and one negative optimum for d

2.2 $z > g$ two positive optima for d

We focus our attention on case 2.2.

2.7.2 A2

In order to analyze the effect of an increase in $x(t)$ on the slope of the saddle path (where we let $x(t) + \Delta x(t) \equiv x > \gamma r^*$), we proceed by calculating the derivative of its slope with respect to x.

$$\frac{\partial\left(\frac{v_{21}}{v_{11}}\right)}{\partial x} = \frac{\partial\left(\frac{v_{21}}{v_{11}}\right)}{\partial \lambda_s}\frac{\partial \lambda_s}{\partial d^*}\frac{\partial d^*}{\partial x}$$

$$= \frac{\left(\frac{v_{21}}{v_{11}}\right)^2}{2}\left(1 + \frac{1}{1 - \frac{2\lambda s}{\varepsilon + r^* - g - \theta d^*}}\frac{1}{\left(\varepsilon + r^* - g - \theta d^* - 2\lambda_s\right)^2}\right)\frac{1}{\eta + (g - r^*)\,\delta/\beta}$$

It is possible to show that an increase in x has the effect of swivelling the saddle path anti-clockwise whenever at least one of these conditions is satisfied:

$$\begin{aligned}
\varepsilon + r^* - g - \theta d^* &> 0 \\
\frac{1}{2\lambda_s - (\varepsilon + r^* - g - \theta d)} &> \frac{\left(\varepsilon + r^* - g - \theta d^* - 2\lambda_s\right)^2}{\varepsilon + r^* - g - \theta d^*} \\
-\left(\varepsilon + r^* - g - \theta d^*\right) &> -2\lambda_s \\
\left(\varepsilon + r^* - g - \theta d^* - 2\lambda_s\right)^3 &> -\left(\varepsilon + r^* - g - \theta d^*\right)
\end{aligned}$$

Since they are very general conditions, we can reasonably assume they are satisfied. Thus, the above considerations suggest that an exogenous demand shock implies a rightward shift in the saddle path and an increase in its slope.

2.7.3 A3

If a steady state exists, then it is a saddle point.

Proof. The condition of existence of the steady state is

$$\frac{\beta}{\varepsilon}(\theta + \varphi) + g - r^* > \sqrt{4\varphi\frac{\beta}{\varepsilon}\frac{\alpha(x - \gamma r^*)}{1 + \alpha\gamma}} \tag{2.25}$$

It is easy to show that condition (2.25) entails

$$\sqrt{4\varphi\frac{\beta}{\varepsilon}\frac{\alpha\left(x-\gamma r^{*}\right)}{1+\alpha\gamma}}>2\varphi d\left(t\right)$$

and, therefore, $(\theta+\varphi)\beta/\varepsilon+g-r^{*}>2\varphi d(t)$. Since determinant $D=\varepsilon\left(r^{*}-g+2\varphi d\left(t\right)\right)-\beta\left(\theta+\varphi\right)$ the inequality $D<0$ is satisfied. Then $\lambda_1\lambda_2<0$ and the steady state is a saddle. ■

2.7.4 A4

The sign of the derivative of the steady-state level of the real exchange rate with respect to φ crucially depends upon a critical value for d. If the external liabilities are less than the minimum of the stationarity schedule for external liabilities, d_m, a marginal increase in φ leads to an appreciation. (however, the currency remains weaker than the level that takes trade into balance). If instead $d(t)>d_m$, a marginal increase in φ always leads to a long run currency depreciation. Given the minimum of the stationarity schedule for external liabilities,

$$d_m=\frac{\beta}{\varepsilon}-\sqrt{\left(\frac{\beta}{\varepsilon}\right)^{2}+\frac{\beta}{\varepsilon\theta}\left(g-\frac{r^{*}+\alpha x}{1+\alpha\gamma}\right)}$$

it is possible to express the same condition by finding a critical value for φ below which a marginal increase in home bias always lead to a currency depreciation, i.e.:

$$\varphi^{c}=\frac{1}{d_m}\left(-\varepsilon q\left(t\right)-\theta d_m+\frac{\alpha\left(x-\gamma r^{*}\right)}{1+\alpha\gamma}\right)$$

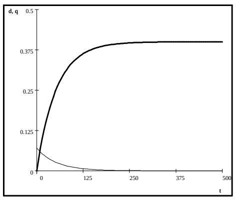
Note finally that if $x(t)=\gamma r^{*}$, the steady state is in the origin and is not affected by $\Delta\varphi$. Indeed:

$$\frac{dd^{*}}{d\varphi}=\frac{d}{d\varphi}\left[\frac{\frac{\beta}{\varepsilon}\left(\theta+\varphi\right)+g-r^{*}-\sqrt{\left(\frac{\beta}{\varepsilon}\left(\theta+\varphi\right)+g-r^{*}\right)^{2}-4\varphi\frac{\beta}{\varepsilon}\frac{\alpha(x(t)-\gamma r^{*})}{1+\alpha\gamma}}}{2\varphi}\right]=0$$

and

$$
\begin{aligned}
\frac{dq_1^*}{d\varphi} &= \frac{d}{d\varphi}\left[\frac{\alpha\left(x\left(t\right)-\gamma r^*\right)}{\varepsilon\left(1+\alpha\gamma\right)}-\frac{\theta+\varphi}{\varepsilon}d_1^*\right] \\
&= 0
\end{aligned}
$$

2.7.5 Simulation results

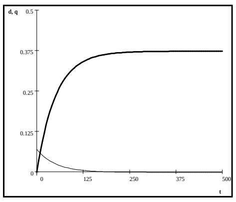


$\varphi = 0, r^* = 0.025$

$g = 0.025$

$x = \gamma r^* + 0.01$

$\eta = 0.025$

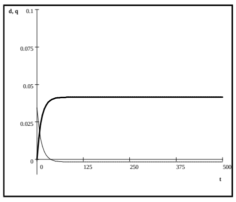


$\varphi = 0.01, r^* = 0.025,$

$g = 0.025$

$x = \gamma r^* + 0.01$

$\eta = 0.025$

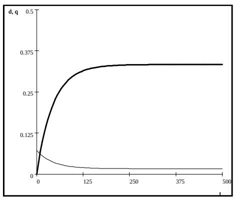


$\varphi = 0.1, r^* = 0.025,$

$g = 0.025$

$x = \gamma r^* + 0.01$

$\eta = 0.025$

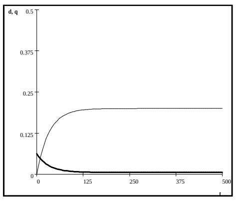


$\varphi = 0, r^* = 0.02,$

$g = 0.025,$

$x = \gamma r^* + 0.01$

$\eta = 0.025$

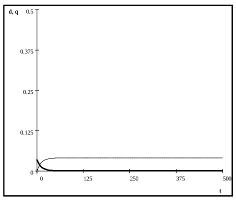


$\varphi = 0.01, r^* = 0.02$

$g = 0.025$

$x = \gamma r^* + 0.01$

$\eta = 0.025$

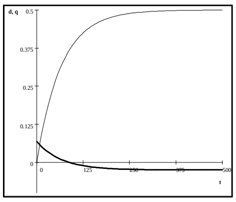


$\varphi = 0.1, r^* = 0.02$

$g = 0.025$

$x = \gamma r^* + 0.01$

$\eta = 0.025$

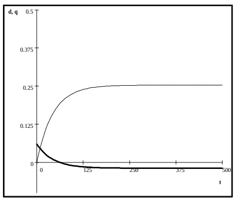


$\varphi = 0, r^* = 0.03$

$g = 0.025$

$x = \gamma r^* + 0.01$

$\eta = 0.025$

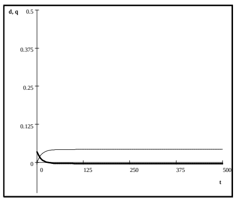


$\varphi = 0.01, r^* = 0.03$

$g = 0.025$

$x = \gamma r^* + 0.01$

$\eta = 0.025$



$\varphi = 0.1, r^* = 0.03$

$g = 0.025$

$x = \gamma r^* + 0.01$

$\eta = 0.025$

Figure 2.5.Change in φ for plausible[11]values of $(r^* - g)$, saddle path

[11]We keep $\alpha = 0.5\beta = 0.1, \gamma = 0.2, \delta = 0.1, (\theta = \frac{\alpha\eta}{1+\alpha\gamma}, \varepsilon = \frac{\alpha\delta}{1+\alpha\gamma})$

r^*	g	Slope for φ=0	Slope for φ=0.001	Slope for φ=0.01	Slope for φ=0.02	Slope for φ=0.1
r^*=0.035	g =0.035	-0.17637	-0.18993	-0.29546	-0.38987	-0.87356
r^*=0.03	g =0.03	-0.17637	-0.18993	-0.29546	-0.38987	-0.87356
r^*=0.025	g =0.025	-0.17637	-0.18993	-0.29546	-0.38987	-0.87356
r^*=0.03	g =0.035	-0.16625	-0.17903	-0.28032	-0.37276	-0.85292
r^*=0.025	g =0.03	-0.16625	-0.17903	-0.28032	-0.37276	-0.852925
r^*=0.02	g =0.025	-0.16625	-0.17903	-0.28032	-0.37276	-0.85292
r^*=0.035	g =0.03	-0.18745	-0.20202	-0.31221	-0.40833	-0.89482
r^*=0.03	g =0.025	-0.18745	-0.20202	-0.31221	-0.40833	-0.89482

Table 2.2. Change in $(r^* - g)$ and saddle path.

x	Slope for φ=0	Slope for φ=0.001	Slope for φ=0.01	Slope for φ=0.02	Slope for φ=0.1
x = γ r^*+0.02	-0.18375	-0.20045	-0.32544	-0.42754	-1.34112
x = γ r^*+0.001	-0.19052	-0.20324	-0.30314	-0.39454	-0.87440
x = γ r^*+0.0005	-0.19069	-0.20331	-0.30270	-0.39385	-0.87331

Table 2.3. Change in exogenous demand shock and saddle path. $r^* = 0.03$; $g = 0.025$

η	Slope for φ=0	Slope for φ=0.001	Slope for φ=0.01	Slope for φ=0.02	Slope for φ=0.1
η=0.025	-0.18745	-0.20324	-0.30314	-0.39454	-0.87440
η=0.027	-0.20190	-0.21426	-0.31201	-0.40207	-0.87859
η=0.03	-0.21838	-0.23027	-0.32503	-0.41318	-0.88485

Table 2.4. Change in wealth effect and saddle path. $r^* = 0.03$; $g = 0.025$; $x = \gamma r^* + 0.001$

Chapter 3

External imbalances and collateral constraints in a two-country model.

3.1 Introduction

During the last decades, the world economy has experienced the emergence of significant external imbalances. In some major economies, in light of the dramatic increase in the accumulation of net external liabilities, policy makers have questioned the sustainability of current trends[1]. Indeed, "There is likely to be a limit to the amount of debt that a country can issue as a result of persistent deficits before investors start to worry about its ability or willingness to repay" (Mervyn King, 2005).

In general equilibrium models, unsustainable trends are generally ruled out by introducing standard no-Ponzi game conditions. However, no-Ponzi game conditions *per se* are not sufficient to ensure creditors against the risk of default and are conditional on future realizations. More restrictive and general conditions have been thus incorporated in models addressing risky environments, such as exogenous debt limits. For instance, thumb rules for solvency have often been imposed to emerging countries and used as a benchmark for sustainability[2].

In the developed world, agents' ability to consume has been increasingly limited by collateral

[1] There is now a very rich and variegated literature on US external imbalances and the sustainability of US external debt. For some discussion on UK external imbalances see among others Iliopulos and Miller (2007).

[2] For some discussion on thumb rules see Williamson (1999) and Miller and Zhang (1999). See also Helmann et al. (2000) for some discussion on capital requirements in banking.

constraints. The increase of collateralized debt cannot but be associated to housing finance liberalization. Indeed, in the past 30 years, the development of financial markets has been associated to liberalization of housing finance in several countries. The process took different forms in different countries;[3] eventually, it allowed households a better access to mortgages credit.[4]

Collateral constraints insure that in case of default, the creditor can repossess (part of) the asset. The existence of collateral constraints creates a link between assets value and aggregate domestic debt; in turn, they create a direct link between fluctuations of real asset prices and debt levels. Notice however that spillovers deriving from housing finance are not only a purely domestic issue. Indeed, financial intermediaries have increasingly converted mortgages in assets. Households' debt has been thus sold abroad in the form of international assets. Therefore, collateral constraints have reinforced the transmission mechanism linking real assets and international asset prices. Notice finally that the value of international (real) assets depends on terms of trade (relative house prices); on the other hand, yields on international assets are affected by exchange rate swings. Collateral constraints have thus also reinforced the mechanism linking real wealth, international capital flows and exchange rates.

In Figure 3.1b we show the trends of current account and house prices for a panel of industrial countries characterized by significant external imbalances. For those countries, current account deficits show a co-movement with house prices, in the last decade: increasing current account deficits (surpluses) are indeed associated to rising (decreasing) house prices.

The linkage between agents' wealth, exchange rates and the direction of international capital flows has been emphasized in the context of currency crises in the emerging world. Krugman (1999) studies the link between currency depreciation and the wealth of collateral-constrained firms in the context of the Asian crisis.[5] He shows how depreciation has a negative (and dramatic) impact on the wealth of firms if external debt is denominated in foreign currency; in turn, as implied by Bernanke and Gertler (1989), the fall in firms' wealth, together with debt

[3] For a survey on the developments in the housing finance, see IMF (2008).

[4] Moreover, by allowing more agents access to collateralized credit, the share of collateral-constraint agents have increasingly boomed. In US, mortgage debt has increased from about 60% at the half of the last century to about 90% – including veichles.

[5] Aghion et al. (2000) provide a supply-driven story with analogous implications.

limits, can lead to a collapse in investments and in aggregate demand.[6]

In our analysis we transfer Krugman's (1999) open-economy "Bernanke-Gertler" effect to a DSGE framework for the analysis of current account dynamics. This mechanism is at the roots of the transmission of various stochastic shocks. We will in particular focus on development affecting agents' wealth (i.e., affecting the housing sector), and their impact on current account and exchange rate dynamics. We will track the response of the economy to demand shocks and the financial liberalization in housing finance. Finally, following Ferrero et al. (2008), we will analyze the response of our two-country world to technology shocks; this will provide a deeper understanding of the mechanisms driving the dynamics of our model.[7]

The role of collateral constraints has been widely analyzed in a closed economy framework. In presence of durable goods, Kiyotaki and Moore (1997) extend the seminal result of Becker (1980) and Becker and Foias (1987) with discounting heterogeneity to the case of collateral constraints. Their analysis shows that the steady-state level of debt (wealth) of impatient (patient) agents is defined by the debt constraint itself; moreover the collateral constraint plays an important role in transmitting the effects of various shocks to other sectors. Indeed, the "financial accelerator mechanism" (see Bernanke et al., 1999) amplifies endogenous developments hitting the credit market. Notice however that when debt contracts are in nominal terms and monetary policy controls interest rates, inflation dynamics amplify demand shocks but dampen supply shocks – working thus as a "financial decelerator" (see Iacoviello 2005). The reason is that, with nominal debt contracts, an increase in inflation has a positive effect on debtors' wealth and a negative one of the wealth of borrowers – with important implications in light of an optimal monetary policy (see Monacelli, 2007).

In this essay, we extend Iacoviello (2005) and Monacelli (2007, 2008) New-Keynesian framework to a two country-world. This framework is very useful to analyze situations where agents are indebted in equilibrium. Indeed, as in Kiyotaki and Moore (1997), discounting heterogeneity and debt limits insure that the collateral constraint is always binding. Moreover, their New-Keynesyan approach allows analyzing the trends of nominal variables and the role of mon-

[6] Caballero and Krishnamurthy (2001) build a model on emerging market crises where they analyze the interactions between domestic and international collateral constraints on firms with limited borrowing capacity.

[7] Iacoviello and Neri (2008) show that technology shocks and housing demand shocks count for one quarter each of the cyclical volatility of US housing investment and prices. Moreover, they show how in recent decades demand factors may have played a more prominent role.

etary policy. Price rigidities and the monetary policy stance could indeed play a role in current account and exchange rate dynamics.[8]

The purpose of our work present some common features with the ones of Callegari (2007) and Punzi (2007).[9] In a busyness cycle framework, Punzi (2007) studies current account and house prices dynamics in presence of heterogeneous agents; similarly, Callegari (2007) focuses on the twin-deficit hypothesis by analyzing the impact of fiscal shocks on US trade balance. However, none of these works incorporate nominal prices and exchange rates. There is thus no scope to analyze the above-discussed link of housing trends and international capital flows with exchange rates. Finally, it is also impossible for them to account for the role of monetary policy nor for price rigidities.

In our two-country world, each economy produces both tradable goods and non-tradable real properties. Tradables and houses are produced by an infinite set of monopolistic firms. Intermediate goods are then sold to final consumers by final retailers who operate in a perfectly competitive environment. We suppose that agents are heterogeneous in their discount factor; in our framework, agents' impatience plays an important role in explaining external debt trends. This hypothesis is not new in international finance.[10] Ghironi et al. (2005) introduce heterogeneous discounting in an overlapping generation framework[11]. More recently, Choi et al. (2008) track the dynamics of US current account introducing endogenous heterogeneous discounting.

Impatient agents are constrained in their access to credit by collateral debt limits. In particular, their ability to borrow is a positive function of the value of their real properties: the greater the value, the better credit access. Domestic borrowing is then sold abroad as an international security to patient agents. Eventually, our world economy is characterized by a positive level of external debt: the stricter the collateral constraints, the lower the steady-state

[8] The IMF suggests in this respect that "..given the imperfect global integration of markets for goods and services and the rigidities that constrain the reallocation of resources to tradable sectors, the redistribution of world spending is likely to require considerable movements in real exchange rates.." (IMF, 2007).

[9] Matsuyama (1990) analyzes the effects of fiscal policy shocks on the current account. In his small-economy setting, an increase in government spending represents a negative income shock for households; it thus dampens their consumption of house services and improves the current account balance. See also İscan (2002) for an empirical analysis on current account and durables.

[10] Our hypothesis is also consistent with Masson et al. (1994) and Henriksen (2002) who relate current account dynamics with demographic factors. Analogously, Chinn and Prasad (2000) find that demographic factors are significant determinant of the current account balance.

[11] For more discussion, see also Buiter (1981) and Weil (1989).

level of debt. Clearly, all shocks that affect house prices require adjusting external debt – and thus, a current account adjustment process. Terms of trade play an important role as a transmission channel of country and sector specific shocks.

The essay is organized as follows. In Section 3.2 we introduce our model and in Section 3.3 we analyze the steady state. In Section 3.4 we focus on the dynamics following shocks hitting the housing market while in Section 3.5 we analyze the behavior of our model in response to productivity shocks. Section 6 comments the main results of our analysis while the Appendix provides analytical details and Figures.

3.2 The model

This model is built on Monacelli (2007, 2008) and Iacoviello (2005) closed economies but is developed in a two-country setting. We consider Home and Foreign respectively (denoted by H and F for simplicity). Both countries are open in every ways but labor. The inhabitants of both countries have same preferences but are heterogeneous in their degree of impatience. More precisely, we assume that the representative inhabitant of country H is more impatient that the one of country F. S/he is not a consumption smoother but her/his desire to consume is limited by a collateral constraint. For simplicity, we will denote the inhabitant of country H as the borrower and the one of country F as the saver.[12]

Durable goods (real properties) and tradable goods are produced in a monopolistic competition framework by domestic and foreign firms; real properties are non-tradable goods and can be used as collateral. Think for instance of houses: leaving tourism a part, houses are generally owned by and sold to residents.[13] Goods are then purchased and sold to final consumers by domestic retailers, in a competitive environment. The representative retailer in the housing sector in country H (in country F), buys Home (Foreign)-produced durables only and sell them to final consumers in country H (country F) only. Analogously, the representative retailer in the tradable sector, buys both Home and Foreign-produced goods to sell them to final consumers in the Home (Foreign) country only. Final consumers enjoy services deriving

[12] Notice however that this will be an endogenous result of the model.
[13] See also Engel and Wang (2008) for some discussion.

from durables[14] and consume tradable goods. Finally, agents can smooth their consumption by exchanging securities on international incomplete markets; debt in country H is subject to a collateral constraint.

3.2.1 Retailers

We suppose that intermediate goods are sold in both countries to final consumers by an infinite set of retailers operating in a competitive environment. Goods markets in each country are segmented into the tradable and real properties sector. We will denote by $j = T$, the representative retailer operating in the tradable sector; $j = n$, the representative retailer operating in the durable sector (real properties). The retailer $j = T$ in country H (in country F) buys Home and Foreign-produced tradables and sell them to the Home (Foreign) market. The retailer $j = n$ buys real properties produced in country H (country F) and sell them to the Home (Foreign) market.

Tradables

Consider first the case of the retailer operating in the tradable sector in country H. S/he has access to both domestic and foreign-produced goods and sell them to H final consumers only. In order to reflect consumer's preferences, we assume that the behavior of retailers is characterized by home bias.[15] Retailers operate in a perfectly competitive environment and their basket of production is the following CES bundle:

$$Y_{T,t} = \left[\alpha^{\frac{1}{\eta}} Y_{h,t}^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} Y_{f,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

where α represents the weight of Home produced goods in consumers' bundles (in presence of home bias, $\alpha > 0.5$) and $\eta > 0$ is the elasticity of substitution between Home and Foreign produced goods. For simplicity, from now on we denote all variables referred to Home (Foreign) produced tradables and prices with the index h (index f). Retailers' demand for respectively

[14] Services deriving from housing are proportional to the stock of houses; moreover, agents start enjoying these services starting from the first period the buy the house.

[15] An alternative way to introduce home bias in our model would be to leave the choice between domestically-produced versus foreign-produced goods to consumers.

Home and Foreign produced goods is the result of an optimization problem. The CES-related price index of tradables is:

$$P_{T,t} = \left[\alpha P_{h,t}^{1-\eta} + (1-\alpha) P_{f,t}^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{3.1}$$

Retailers' intermediate demand for Home produced and Foreign produced goods ($Y_{h,t}$ and $Y_{f,t}$, respectively) is:

$$Y_{h,t} = \alpha Y_{T,t} \left(\frac{P_{h,t}}{P_{T,t}}\right)^{-\eta} \tag{3.2}$$

$$Y_{f,t} = (1-\alpha) Y_{T,t} \left(\frac{P_{f,t}}{P_{T,t}}\right)^{-\eta} \tag{3.3}$$

In country F, retailers in the tradable sector behave symmetrically. This implies that the weight of Foreign produced goods on country F CES production bundle is the same as the one of Home produced goods in country H[16], i.e.:

$$Y_{T,t}^{*} = \left[(1-\alpha)^{\frac{1}{\eta}} Y_{h,t}^{*\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} Y_{f,t}^{*\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{3.4}$$

Profit maximization implies that retailers' intermediate demand for foreign and domestic goods respectively in country F is:

$$Y_{h,t}^{*} = (1-\alpha) Y_{T,t}^{*} \left(\frac{P_{h,t}^{*}}{P_{T,t}^{*}}\right)^{-\eta} \tag{3.5}$$

$$Y_{f,t}^{*} = \alpha Y_{T,t}^{*} \left(\frac{P_{f,t}^{*}}{P_{T,t}^{*}}\right)^{-\eta} \tag{3.6}$$

Notice however that retailers also need to choose amongst the different (infinite) varieties of heterogeneous domestic and foreign goods, i. We suppose that the Home (Foreign)-produced basket of the representative retailer is in turn a CES bundle of a continuum of infinite varieties of goods, i.

[16] The corresponding price index is: $P_{T,t}^{*} = \left[(1-\alpha) P_{h,t}^{*1-\eta} + \alpha P_{f,t}^{*1-\eta}\right]^{\frac{1}{1-\eta}}$

We reasonably assume that the elasticity of substitution amongst Home (Foreign) produced goods ε, is greater than the one between Home produced and Foreign goods, η. Retailers' intermediate demand for a single variety of Home produced (Foreign) good is thus:

$$Y_{h,t}(i) = Y_{h,t}\left(\frac{P_{h,t}(i)}{P_{h,t}}\right)^{-\varepsilon} \tag{3.7}$$

$$Y_{f,t}(i) = Y_{f,t}\left(\frac{P_{f,t}(i)}{P_{h,t}}\right)^{-\varepsilon} \tag{3.8}$$

where $\varepsilon > 1$[17]. Analogously, in the rest of the world:

$$Y^*_{h,t}(i) = Y^*_{h,t}\left(\frac{P^*_{h,t}(i)}{P^*_{h,t}}\right)^{-\varepsilon} \tag{3.9}$$

$$Y^*_{f,t}(i) = Y^*_{f,t}\left(\frac{P^*_{f,t}(i)}{P^*_{f,t}}\right)^{-\varepsilon} \tag{3.10}$$

Durables

Consider now the housing sector. The representative retailer in country H chooses a set of houses amongst an infinite continuum of domestically produced real properties. Her/his demand for each differentiated good, i, is the result of profit maximization in a competitive environment [18], i.e.:

$$Y_{n,t}(i) = \left(\frac{P_{n,t}(i)}{P_{n,t}}\right)^{-\varepsilon} Y_{n,t} \tag{3.11}$$

where ε is the elasticity of substitution between single goods, i .[19] Analogously, in the Foreign country, the intermediate demand for real properties of the representative retailer is:

[17] In order to keep countries H and F as symmetric as possible, we assume identical elasticities of substitution across countries.

[18] Where the CES production bundle of the retailer is: $Y_{n,t} \equiv \left(\int_0^1 Y_{n,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} d_i\right)^{\frac{\varepsilon}{\varepsilon-1}}$. The associated price index is: $P_{n,t} \equiv \left(\int_0^1 P_{nj,t}(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}$

[19] For simplicity, we assume that the elasticity of substitution between the infinite varieties of Home (Foreign) produced goods is the same for both sectors.

$$Y^*_{n,t}(i) = \left(\frac{P^*_{n,t}(i)}{P^*_{n,t}}\right)^{-\varepsilon} Y^*_{n,t} \tag{3.12}$$

3.2.2 Optimal consumption in country H

Consider now the representative inhabitant of country H. His/her utility is a positive function of her/his basket of consumption, C_t and a negative function of her/his labor effort (i.e., her/his supply of labor, N_t) i.e.[20]:

$$\max E_0 \left\{ \sum_{t=0} \beta^t U(C_t, N_t) \right\}$$

where β is the borrower's discount factor. We do not introduce explicitly money in the utility function and we use it as the *numéraire* of our cashless economy *à la* Woodford.

The representative borrower consumes a bundle that is a CES composite of tradables and services deriving from the stock of real properties. For simplicity, we assume that agents start enjoying services deriving from durables in the same period they purchase them and are proportional to the stock of houses.[21] The borrower consumption basket is thus:

$$C_t = \left[\gamma^{\frac{1}{\theta}} C_{T,t}^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} C_{n,t}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$$

where γ is the weight of tradables in the basket and $\theta \geqslant 0$ is the elasticity of substitution between durable services and tradables. The following assumption on preferences always needs to hold in both countries:

Assumption 1 (preferences) $U_k \in C^2$, $U_{kC} > 0$, $U_{kN} < 0$, $U_{kCC} < 0$, $U_{kCC}U_{kNN} > U^2_{kCN}$ for every (C_k, N_k) such that $C_k, N_k > 0$, $k =$borrower, saver. Also, Inada conditions for consumption hold.

The individual budget constraint in real terms of tradable consumption is[22]:

[20] In our economy all agents have the same preferences and maximize an unility function that we will assume in the numerical simulations having the following form: $U_t = \ln C_t - \left(\frac{v}{1+\varphi}\right) N_t^{1+\varphi}$

[21] We assume also that agents cannot rent/lend houses. This implies that CPI inflation corresponds to aggregate inflation in the tradable goods sector.

[22] The individual budget constraint in nominal terms is:

$$C_{T,t} + x_t \left(C_{n,t} - (1-\delta) C_{n,t-1}\right) + R_{t-1} \frac{b_{t-1}}{\pi_{T,t}} \leq b_t + \frac{W_t N_t}{P_{T,t}} + \sum \frac{\Gamma}{P_{T,t}} \quad (3.13)$$

where $C_{T,t}$ represents tradable consumption, $P_{n,t}\left(C_{n,t} - (1-\delta) C_{n,t-1}\right)$ is the cost of durable expenditure in period t; b are net external liabilities in real terms of tradable consumption[23], where $B \equiv D - qD^*$ are net external liabilities in nominal terms, D are home-currency domestic securities, q is the nominal price of Foreign currency in terms of Home currency and D^* are foreign-currency Foreign securities[24]. Notice also that $x_t \equiv \frac{P_{n,t}}{P_{T,t}}$ is the relative price of real properties and $\pi_{T,t} \equiv \frac{P_{T,t}}{P_{T,t-1}}$ is the aggregate inflation rate in the tradable sector[25]. In practice, in each period the borrower buys tradables, C_T, and real properties; s/he pays the interests on her/his debt – R is the gross nominal interest rate factor. S/he enjoys resources coming from foreign borrowing, B, labor income, WN and profits, Γ coming from their firms (operating in a monopolistic competition framework). Labor is assumed mobile across sectors but not across countries; therefore, the wage is the same in each sector but not necessarily in each country.

Agents relative impatience (see Assumption 2) implies that the representative agent in country H is not a consumption smoother, i.e., s/he does not have an intertemporal budget constraint (for more discussion, see the following)[26]. Assuming that the inhabitants of country H are more impatient than the ones of country F is clearly a simplification. However, this simplification allows us to consider international differences in countries' discount factors at an aggregate level;[27] also, it allows us investigating the implications arising from positive debt levels in equilibrium. Buiter (1981), Ghironi et al. (2005) and Weil (1989) introduce heterogeneous discount rates in a framework of overlapping-generations; in Choi et al. (2008) heterogeneous discount rates are endogenous.

$$\begin{aligned} & P_{T,t}C_{T,t} + P_{n,t}\left(C_{n,t} - (1-\delta) C_{n,t-1}\right) + R_{t-1}B_{t-1} \\ \leq \; & B_t + W_t N_t + \sum \Gamma \end{aligned}$$

The budget constraint is assumed to hold with equality around the deterministic steady state.

[23] Notice that $b_{t-1} = \frac{B_{t-1}}{P_{T,t-1}}$

[24] See the Appendix for all details concerning the optimization program of the consumer.

[25] The inflation rate in the durable sector is defined as $\pi_{n,t} \equiv \frac{P_{n,t}}{P_{n,t-1}}$, while $\pi_{h,t} \equiv \frac{P_{h,t}}{P_{h,t-1}}$

[26] See also Iacoviello (2005).

[27] In a two-country framework, this reduces to the case where the representative agent of one country is less patient than the one of another country.

Borrowers' capacity to obtain credit is limited by a collateral constraint. We suppose that households' debt is constrained to be a share of the value of their durables (real properties), and debt contracts are denominated in nominal terms, i.e.:

$$b_t \leq (1-\chi)\, C_{n,t} x_t \tag{3.14}$$

where χ is the fraction of durables that cannot be used as a collateral and can be interpreted as the inverse of the loan-to-value ratio: the larger χ, the more stringent the constraint. For simplicity, we assume χ to be an exogenous parameter of our model. The role of collateral constraints and the implications of their structure has been recently analyzed in a New-Keynesyan framework by Calza *et al.* (2006) in a closed economy framework.[28] Eventually, collateral constraints allow agents a better access to credit. Indeed, they partially ensure the creditor against the risk of default: in case of default, the creditor can always repossess (part of) the asset.[29]

In our two-country world, constraint (3.14) reduces to a limit on international borrowing. This should not surprise the reader. Since we aim at analyzing current account dynamics, we are interested in the behavior of aggregate variables, and in the dynamics of flows (of goods and financial capital) between countries. In aggregate, the sum of domestically traded assets and liabilities in each country is equal to zero. Thus, if indebted, our representative agent of each country cannot but be indebted towards his foreign counterpart only. Indeed, thanks to the globalized structure of financial systems, mortgages can be easily converted in international assets. Our representative agent in each country can thus act as a financial intermediary and sold her/his (collateralized) debt abroad. In this vein, collateral constraints (and their impact) are transferred to an international dimension.

Notice finally that (3.14) implies also that an increase in the relative price of real properties allows agents to increase their level of debt.

[28] For more discussion see also Monacelli (2006, 2007), Iacoviello (2005) and Campbell and Hercowitz (2006).

[29] In Bernanke and Gertler (1989) they are justified by the presence of private information and limited liability. In Kiyotaki and Moore (1997) they are the response to problems of enforcing contracts.

The first order conditions of borrowers' optimization program are:

$$\frac{-U_{N,t}}{U_{T,t}} = \frac{W_t}{P_{T,t}} \tag{3.15}$$

$$x_t U_{T,t} = U_{n,t} + \beta(1-\delta) E_t \{U_{T,t+1} x_{t+1}\} + U_{T,t}\psi_t (1-\chi) x_t \tag{3.16}$$

$$\psi_t = 1 - \beta E_t \left\{ \frac{R_t}{\pi_{T,t+1}} \frac{U_{T,t+1}}{U_{T,t}} \right\} \tag{3.17}$$

Equation (3.15) represents a standard consumption/leisure arbitrage. Equation (3.16) represents the intertemporal demand for tradable consumption relatively to durable-services consumption. In equilibrium, the value of the utility deriving to the borrower from present consumption of tradables needs to equal the value of direct utility deriving from the direct housing services plus the value of their indirect utility, i.e.: i) the utility deriving from the possibility of selling real properties and buying durable consumption in future, $\beta(1-\delta) E_t \{U_{T,t+1} x_{t+1}\}$ and ii) the marginal utility stemming from relaxing the collateral constraint, and consuming additional non-durable goods, $U_{T,t}\psi_t (1-\chi) x_t$.

Equation (3.17) represents a modified Euler equation of country H – where $\lambda_t \psi_t$ is the Lagrangian multiplier associated to the collateral constraint and λ_t the one associated to the budget constraint. Clearly, it reduces to the standard Euler equation whenever ψ_t =0. ψ_t represents the marginal value of additional debt. If $\psi_t > 0$, the marginal utility of current consumption exceeds the gain of shifting intertemporally one unit of consumption (i.e., savings): $U_{T,t} > \beta E_t \left\{ \frac{R_t}{\pi_{T,t+1}} U_{T,t+1} \right\}$.[30] By marginally relaxing the collateral constraint one could have access to more current consumption and increase her/his utility.

Finally, one can rewrite (3.16) as:

$$\frac{U_{n,t}}{U_{T,t}} = x_t (1 - \psi_t (1-\chi)) - \beta(1-\delta) E_t \left\{ \frac{U_{T,t+1}}{U_{T,t}} x_{t+1} \right\} \tag{3.18}$$

Notice that the RHS of equation (3.18) represents the user cost of durables in terms of non-durables at time t. It represents the price you pay for the flow of consumption services from durables during the period; the cost is obviously a positive function of the interest rate and the relative price of durables (for ψ_t fixed). By substituting (3.17) in (3.18) and log-linearizing

[30] ψ can also be interpreted as the price of an asset; indeed, it is tied to a payoff that depends on the deviation from the Euler equation – see Monacelli (2007).

(3.18), it is possible to isolate the effect of ψ on the user cost, during the dynamics around the steady state, as shown in the Appendix (its impact on the steady state is also discussed in the Appendix). The impact of a variation in ψ on the user cost crucially depends both on the structure of the collateral constraint and the parametrization. In our framework, it is generally negative. This implies that a decrease in the marginal utility of borrowing makes houses less useful and entails a substitution in favour of tradables.[31]

3.2.3 Exchange rates and terms of trade

In presence of home bias, the law on one price only holds for the single basket of Foreign produced and Home produced tradables, separately. In practice:

$$P_{h,t} = q_t P^*_{h,t}$$

and

$$P_{f,t} = q_t P^*_{f,t}$$

where q is the nominal exchange rate (the Home-currency price of Foreign currency) and $\alpha > \frac{1}{2}$ implies $P_{T,t} \neq q_t P^*_{T,t}$.

In addition, in equilibrium, the following relationship between Home and Foreign net external liabilities always needs to hold:

$$B_t = q_t B^*_t \tag{3.19}$$

where B is Home-currency net external debt in nominal terms and B^*[32] are lenders' net external assets in Foreign currency. Obviously, if $B > 0$ borrowers are net debtor and savers are net lenders.

In our two-country simple world, houses cannot be rent. This implies that the CPI price index coincides with the aggregate price index of tradable goods, P_T.[33] It is thus straightforward to calculate the CPI real exchange rate of country H as:

[31] Having said that, for very small depreciation rates, we recover Monacelli (2008) result in presence of a (slightly) different collateral constraint. See the Appendix.

[32] Clearly, $B^* = \frac{D}{q} - D^*$

[33] Indeed, the CPI basket does not include the price of houses.

$$\epsilon_t = q_t \frac{P^*_{T,t}}{P_{T,t}}$$

For simplicity, from now on when referring the real exchange rate of H we will consider the CPI based real exchange rate.[34] Notice finally that in absence of home bias, the CPI-based real exchange rate is constant in all periods; this would certainly be at odds with reality (see Figure 3.1a).

We finally introduce country H terms of trade and we define them in the following way:

$$S_t = \frac{P_{f,t}}{P_{h,t}} = \frac{P^*_{f,t}}{P^*_{h,t}} \tag{3.20}$$

Symmetrically, country F terms of trade are thus:

$$S^*_t = \frac{P_{h,t}}{P_{f,t}} = \frac{P^*_{h,t}}{P^*_{f,t}} = S_t^{-1}$$

The bond market-clearing condition (3.19) can be rewritten as a function of terms of trade, i.e.:

$$b^*_t = b_t \left(\frac{(1-\alpha) S_t^{1-\eta} + \alpha}{\alpha S_t^{1-\eta} + 1 - \alpha} \right)^{\frac{1}{1-\eta}} \tag{3.21}$$

3.2.4 Optimal consumption in country F

We consider now the representative agent of country F. We suppose that agents in country F are more patient than agents in country H. Thus, the discount rate of the borrower is strictly lower than the one of country F representative agent (the saver). Savers maximizes the following utility function:

$$\max E_0 \left\{ \sum_{t=0} \mu^t U^* \left(C^*_t, N^*_t \right) \right\}$$

where μ is their discount factor, N^*_t is labor effort and C^*_t is a CES composite good of tradables and services arising from durables. For simplicity, from now on, all variables referring to the

[34] Analogously, in presence of durable goods, Engel and Wang (2008) build a non-utility based consumption price index. This index is calculated as a weighted average of different prices subindexes and is used to compute the real exchange rate of their economy.

foreign country (and currency) will be indexed by an asterisk. Also, Assumption 2 always need to hold:

Assumption 2 (discounting) $\beta,\ \mu \in (0,1)\,;\beta < \mu$.

Assumption 2 is crucial in our model and has important implications. We will analyze all implications for the steady state in Section 3.3. Notice for the moment that the saver is a consumption smoother and has thus an intertemporal budget constraint.

Savers' consumption basket is composed as the one of borrowers, i.e.:

$$C_t^* = \left[\gamma^{\frac{1}{\theta}} C_{T,t}^{*\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} C_{n,t}^{*\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$$

where C_n^* are services deriving from real properties in the Foreign country and C_T^* is a basket of tradables. The budget constraint of the F-agent in real terms of tradable consumption is[35]:

$$C_{T,t}^* + x_t^* \left(C_{n,t}^* - (1-\delta)\, C_{n,t-1}^*\right) - R_{t-1}^* \frac{b_{t-1}^*}{\pi_{T,t}^*} \leq -b_t^* + \frac{W_t^* N_t^*}{P_{T,t}^*} + \sum_i \frac{\Gamma^*}{P_{T,t}^*} \tag{3.22}$$

where W_t^* are foreign-currency wages in the Foreign country, R^* are nominal interest rates in the Foreign country and Γ^* are profits. Finally, $B^* \equiv \frac{D}{q} - D^*$ are Foreign net external assets in Foreign currency and b_t^* are net external assets in real terms of tradables. Finally, we introduce a no-Ponzi game condition on net international assets[36]:

$$\lim_{i\to\infty} E_t \frac{b_{t+i}^*}{\prod\limits_{z=1}^{i} R_z^*} \leq 0 \tag{3.23}$$

[35]It holds with equality around a deterministic steady state.

[36]This condition does not bind. Analogously, given lenders' patience, any collateral constraint on debt would not bind (see the discussion in the following sections).

The first order conditions of savers' optimization program are:

$$\frac{-U^*_{N,t}}{U^*_{T,t}} = \frac{W^*_t}{P^*_{T,t}} \tag{3.24}$$

$$U^*_{T,t}x^*_t = U^*_{n,t} + \mu(1-\delta)E_t\left\{x^*_{t+1}U^*_{T,t+1}\right\} \tag{3.25}$$

$$U^*_{T,t} = \mu E_t\left\{U^*_{T,t+1}\frac{R^*_t}{\pi^*_{T,t+1}}\right\} \tag{3.26}$$

Equation (3.24) states the standard arbitrage condition between leisure and consumption and equation (3.25) is the intertemporal demand equation for durables *versus* nondurable goods. Equation (3.26) is a standard Euler equation.

Finally, we need to introduce the relation linking interest rates in country H and in country F. In order to obtain it, we substitute for gross external assets and liabilities in the budget constraint of country H or F (remember that $B \equiv D - qD^*$). Rewriting the budget constraint in real terms of tradable consumption and re-calculating the first order conditions for the borrower and/or the lender, one can easily find the needed condition. It is possible to show (see the Appendix for all details) that the following modified uncovered interest parity condition needs to hold:

$$R_t = E_t\left\{\frac{q_{t+1}}{q_t}\right\}R^*_t \tag{3.27}$$

The absence of arbitrage possibilities between domestic and foreign assets implies that the marginal utility of investing in Home assets is equal to the one agents obtain by investing in Foreign assets. Notice finally that given the stochastic setup of our framework and the assumption of incomplete markets, the uncovered interest parity condition only holds in expectations.

3.2.5 Production

We now set the structure of production in our two-country world. For simplicity we suppose that labor is homogeneous and mobile across sectors in the same country – but not around the world. We assume also that the representative agent in each country is also the owner of representative firms in each country. Markets in each country are segmented into tradables and durables (real properties). Firms in both sectors operate in a monopolistic competition environment and are characterized by the good they produce.

Real properties' producers at Home (in the Foreign country) only sell their goods to Home (Foreign) markets while tradables' producers sell them to retailers of both countries. We suppose that there are i firms producing i non perfectly substitutable durables (tradable goods). Each firm is characterized by a production function F, which depends on labor, N_l and a productivity shifter A_l, which is common for all firms within the same sector, $l = h, n \;\; (f, n)$. The following proposition needs to hold:

Assumption 3 (technology): F is homogeneous of degree 1 with $F \in C^2, F_N > 0, F_{NN} \leq 0$. Moreover $F(0) = 0$, $\lim_{N \to 0} F'(N) = +\infty$, $\lim_{N \to \infty} F'(N) = 0$.

Tradables

We first focus the attention on the tradable sector in country H. Firm i production function is consistent with Assumption 3 and is defined for simplicity as:

$$F_{i,t}(N_{h,t}(i)) = A_{h,t} N_{h,t}(i)$$

Firms maximize the profit function:

$$E_0 \left\{ \sum_{t=0}^{\infty} \Lambda_{0,t} \left(F_{i,t}(N_{h,t}(i)) P_{h,t}(i) - W_t N_{h,t}(i) - \frac{\omega_T}{2} \left(\frac{P_{h,t}(i)}{P_{h,t-1}(i)} - 1 \right)^2 P_{h,t} \right) \right\}$$

given retailers' demand functions. Λ is the borrower's stochastic discount factor[37] and:

$$\Lambda_{t,t+1} \equiv \frac{\Lambda_{0,t+1}}{\Lambda_{0,t}} \equiv \beta E_t \left\{ \frac{1}{1 - \psi_t} \frac{\lambda_{t+1}}{\lambda_t} \frac{P_{T,t}}{P_{T,t+1}} \right\}$$

where λ is the borrower's Lagrangian multiplier (i.e., the marginal utility of income) of the representative consumer in country H and

$$\frac{\omega_T}{2} \left(\frac{P_{h,t}(i)}{P_{h,t-1}(i)} - 1 \right)^2 P_{h,t}$$

[37] Profits here are in nominal terms while the consumer maximization program is in real terms. That's why we need to incorporate prices also.

are the firm's costs associated to adjusting prices (menu costs); following Rotemberg and Woodford (1997), we assume that adjustment costs are quadratic. In practice, each period firms need to optimally balance the costs arising from resetting prices and the costs associated to deviating from optimality.

Analogously, the stochastic discount factor of firms in country F is:

$$\Lambda^*_{t,t+1} \equiv \frac{\Lambda^*_{0,t+1}}{\Lambda^*_{0,t}} \equiv \mu E_t \left\{ \frac{\lambda^*_{t+1}}{\lambda^*_t} \frac{P^*_{T,t}}{P^*_{T,t+1}} \right\}$$

where Λ^* is lenders' stochastic discount factor and λ^* is the Lagrangian multiplier associated to the saver's budget constraint.

In both countries firms choose their optimal sequence $\{N_{h,t}(i), P_{h,t}(i)\}, \left(\left\{N^*_{f,t}(i), P^*_{f,t}(i)\right\}\right)$. Nominal and real marginal costs in H (MC and mc, respectively) are:

$$\begin{aligned} MC_{h,t} &= \frac{W_t}{A_{h,t}} \\ mc_{h,t} &= \frac{W_t}{P_{h,t} A_{h,t}} \end{aligned} \tag{3.28}$$

In a symmetric equilibrium: $P_{h,t}(i) = P_{h,t}$. We can thus simply solve for optimal prices. We obtain the following New Keynesian Phillips Curve (NKPC):

$$\begin{aligned} \omega_T (\pi_{h,t} - 1) \pi_{h,t} &= F_{h,t}(N_{h,t}) \varepsilon \left[\frac{(1-\varepsilon)}{\varepsilon} + mc_{h,t} \right] \qquad (3.29) \\ &+ \omega_T \beta E_t \left\{ \left(\frac{1}{1-\psi_t} \right) \frac{U_{T,t+1}}{U_{T,t}} \left[\frac{\alpha + (1-\alpha) S_t^{1-\eta}}{\alpha + (1-\alpha) S_{t+1}^{1-\eta}} \right]^{\frac{1}{1-\eta}} (\pi_{h,t+1} - 1) \pi_{h,t+1} \right\} \end{aligned}$$

The standard optimization program of the representative agent implies that in equilibrium there cannot be gains in exchanging leisure with consumption; the non-arbitrage condition leisure/consumption (3.15) needs to hold. Condition (3.15) can be here rewritten as:

$$\frac{-U_{N,t}}{U_{T,t}} = \frac{W_t}{P_{h,t} \left[\alpha + (1-\alpha) S_t^{1-\eta} \right]^{\frac{1}{1-\eta}}}$$

therefore, when we substitute it in (3.28), we obtain:

$$mc_{h,t} = \frac{1}{A_{h,t}} \frac{-U_{N,t}}{U_{T,t}} \left[\alpha + (1-\alpha) S_t^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{3.30}$$

Clearly, terms of trade affects the Phillips curve both in the form of the marginal cost, and through the discount factor. By incorporating (3.30) and the relation $F_{h,t}(N_{h,t}) = A_{h,t}N_{h,t}$ in the above Phillips curve, we obtain:

$$\begin{aligned} \omega_T (\pi_{h,t} - 1) \pi_{h,t} &= A_{h,t}N_{h,t}\varepsilon \left[\frac{(1-\varepsilon)}{\varepsilon} + \frac{1}{A_{h,t}} \frac{-U_{N,t}}{U_{T,t}} \left(\alpha + (1-\alpha) S_t^{1-\eta}\right)^{\frac{1}{1-\eta}}\right] \\ &+ \omega_T \beta E_t \left\{ \left(\frac{1}{1-\psi_t}\right) \frac{U_{T,t+1}}{U_{T,t}} \left[\frac{\alpha + (1-\alpha) S_t^{1-\eta}}{\alpha + (1-\alpha) S_{t+1}^{1-\eta}}\right]^{\frac{1}{1-\eta}} (\pi_{h,t+1} - 1) \pi_{h,t+1} \right\} \end{aligned} \tag{3.31}$$

Without price rigidities, the real marginal cost is constant at the mark-up. Notice also that in the tradable sector, terms of trade create a wedge between the rate of substitution between consumption and leisure on the one hand, and the marginal product of labor on the other. The real marginal cost is directly affected by movements in terms of trade. This creates a scope for policy intervention whenever the policy-maker aims at optimal policies (for some discussion see Faia and Monacelli, 2007).

Analogous considerations apply for country F. Marginal costs are:

$$mc^*_{f,t} = \frac{1}{A^*_{f,t}} \frac{-U^*_{N,t}}{U^*_{T,t}} \left[\alpha + (1-\alpha) S_t^{-1+\eta}\right]^{\frac{1}{1-\eta}} \tag{3.32}$$

and the NKPC is:

$$\begin{aligned} \omega_T \left(\pi^*_{f,t} - 1\right) \pi^*_{f,t} &= A^*_{f,t}N^*_{f,t}\varepsilon \left[\frac{(1-\varepsilon)}{\varepsilon} + \left(\alpha + (1-\alpha) S_t^{-1+\eta}\right)^{\frac{1}{1-\eta}} \frac{-U^*_{N,t}}{A^*_{f,t}U^*_{T,t}}\right] \\ &+ \omega_T \mu E_t \left\{ \frac{U^*_{T,t+1}}{U^*_{T,t}} \left[\frac{\alpha + (1-\alpha) S_t^{-1+\eta}}{\alpha + (1-\alpha) S_{t+1}^{-1+\eta}}\right]^{\frac{1}{1-\eta}} \left(\pi^*_{f,t+1} - 1\right) \pi^*_{f,t+1} \right\} \end{aligned} \tag{3.33}$$

Durables

The price dynamics in the housing sector can be easily individuated by following the same lines of the previous section. The New Keynesian Phillips curve for durables (real properties) is:

$$\begin{aligned} & \omega_n(\pi_{n,t}-1)\pi_{n,t} \\ = \; & A_{n,t}N_{n,t}\varepsilon\left[\frac{(1-\varepsilon)}{\varepsilon}+mc_{n,t}\right]+\omega_n\beta E_t\left\{\frac{1}{1-\psi_t}\frac{\Lambda_{0,t+1}}{\Lambda_{0,t}}(\pi_{n,t+1}-1)\pi_{n,t+1}\frac{P_{n,t+1}}{P_{n,t}}\right\} \end{aligned}$$

where, again:

$$\Lambda_{t,t+1}=\frac{\Lambda_{0,t+1}}{\Lambda_{0,t}}=\beta E_t\left\{\frac{1}{1-\psi_t}\frac{\lambda_{t+1}}{\lambda_t}\frac{P_{T,t}}{P_{T,t+1}}\right\}$$

and thus:

$$\begin{aligned} & \omega_n(\pi_{n,t}-1)\pi_{n,t} \\ = \; & A_{n,t}N_{n,t}\varepsilon\left[\frac{(1-\varepsilon)}{\varepsilon}+mc_{n,t}\right]+\omega_n\beta E_t\left\{\frac{1}{1-\psi_t}\frac{U_{T,t+1}}{U_{T,t}}\frac{x_{t+1}}{x_t}(\pi_{n,t+1}-1)\pi_{n,t+1}\right\} \end{aligned}$$

Also, given the arbitrage consumption-leisure, we can rewrite firms' marginal costs in the housing sector as:

$$mc_{n,t}=\frac{1}{A_{n,t}x_t}\frac{-U_{N,t}}{U_{T,t}} \tag{3.34}$$

Notice that there is a wedge between the ratio of marginal utilities and the marginal cost. This wedge is created by a variation in the relative price, x. When the monetary authority aims at implementing an optimal policy, the presence of this wedge leaves the policy maker a scope for policy intervention (for some discussion see Monacelli, 2007).

Incorporating (3.34) in the above New-Keynesian Phillips curve, we obtain:

$$\begin{aligned} \omega_n(\pi_{n,t}-1)\pi_{n,t} \; = \; & A_{n,t}N_{n,t}\varepsilon\left[\frac{(1-\varepsilon)}{\varepsilon}+\frac{1}{x_t}\frac{-U_{N,t}}{U_{T,t}A_{n,t}}\right] \\ & +\omega_n\beta E_t\left\{\left(\frac{1}{1-\psi_t}\right)\frac{U_{T,t+1}}{U_{T,t}}\frac{x_{t+1}}{x_t}(\pi_{n,t+1}-1)\pi_{n,t+1}\right\} \end{aligned} \tag{3.35}$$

Analogously, the NKPC in country F is:

$$\omega_n \left(\pi^*_{n,t} - 1\right) \pi^*_{n,t} = A^*_{n,t} N^*_{n,t} \varepsilon \left[\frac{(1-\varepsilon)}{\varepsilon} + \frac{1}{x^*_t} \frac{-U^*_{N,t}}{U^*_{T,t} A^*_{n,t}} \right] + \omega_n \mu E_t \left\{ \frac{U^*_{T,t+1}}{U^*_{T,t}} \frac{x^*_{t+1}}{x^*_t} \left(\pi^*_{n,t+1} - 1\right) \pi^*_{n,t+1} \right\} \tag{3.36}$$

where real marginal costs are:

$$mc^*_{n,t} = \frac{1}{x^*_t} \frac{-U^*_{N,t}}{U^*_{T,t} A^*_{n,t}} \tag{3.37}$$

Notice that we have *a priori* allowed for the possibility of price rigidities in the housing sector. Having said that, we will reasonably assume in our benchmark simulations that house prices are not rigid. Indeed, as Iacoviello and Neri (2008)[38] remark, most houses are priced for the first time when they are sold. Moreover, since each house is a very expensive good, in case menu costs had a significant fixed component, the incentive to renegotiate its price would predominate. [39]

3.2.6 Market clearing

Home country

For markets to be cleared in country H, total purchases of real properties need to equal the total domestic production; they also need to account for the costs of price rigidities. We remind the reader that in this model real properties are non-tradable goods. Thus:

$$A_{n,t} N_{n,t} = C_{n,t} - (1-\delta) C_{n,t-1} + \frac{\omega_n}{2} \left(\pi_{n,t} - 1\right)^2$$

Given that labor is not mobile across countries, labor market clearing implies:

$$N_{n,t} + N_{h,t} = N_t \tag{3.38}$$

[38] See also Barsky, House and Kimball (2007).

[39] Our simplified framework does not allow to model phenomena related to asset prices bubbles. Further research should focus on possible ways to introduce price bubbles in the housing sector.

Therefore:

$$A_{n,t}N_{n,t} = C_{n,t} - (1-\delta)\, C_{n,t-1} + \frac{\omega_n}{2}\left(\pi_{n,t} - 1\right)^2 \tag{3.39}$$

Focus now on the Home sector of tradables. Market clearing requires:

$$A_{h,t}N_{h,t} = C_{h,t} + C^*_{h,t} + \frac{\omega}{2}\left(\pi_{h,t} - 1\right)^2$$

where C_h and C^*_h are consumption levels of Home tradables at Home and in the Foreign country, respectively. Notice that local retailers of tradables in country H operate in a perfectly competitive environment and only sell their products to Home inhabitants (in practice, they simply act as aggregators). Therefore, the market of Home-produced tradables clears when the total amount of produced goods equals the aggregate demand of retailers. In practice, $C_{h,t} = Y_{h,t},\;\; C^*_{h,t} = Y^*_{h,t},\, C_{T,t} = Y_{T,t}$. Recalling that retailers' demand for domestically and foreign produced goods are respectively, $Y_{h,t} = \alpha Y_{T,t}\left(\frac{P_{h,t}}{P_{t,t}}\right)^{-\eta}$ and $Y_{f,t} = (1-\alpha)\, Y_{T,t}\left(\frac{P_{f,t}}{P_{t,t}}\right)^{-\eta}$ the market clearing condition for the tradable sector can be rewritten first as:

$$A_{h,t}N_{h,t} = \alpha Y_{T,t}\left(\frac{P_{h,t}}{P_{T,t}}\right)^{-\eta} + (1-\alpha)\, Y^*_{T,t}\left(\frac{P^*_{h,t}}{P^*_{T,t}}\right)^{-\eta} + \frac{\omega}{2}\left(\pi_{h,t} - 1\right)^2$$

and then as a function of terms of trade, i.e.:

$$\begin{aligned} A_{h,t}N_{h,t} \;\;&=\;\; \alpha C_{t,t}\left[\alpha + (1-\alpha)\, S_t^{1-\eta}\right]^{\frac{\eta}{1-\eta}} \\ &\quad + (1-\alpha)\, C^*_{t,t}\left[(1-\alpha) + \alpha S_t^{1-\eta}\right]^{\frac{\eta}{1-\eta}} + \frac{\omega_T}{2}\left(\pi_{h,t} - 1\right)^2 \end{aligned} \tag{3.40}$$

Country F

Given the symmetric structure of our world economy, clearing conditions for country F have a symmetric structure, relatively to the ones of country H. Market clearing conditions for country F are listed in the following.

Labor market clearing requires:

$$N^*_{n,t} + N^*_{f,t} = N^*_t \tag{3.41}$$

Market clearing in the non-tradable sector requires:

$$A^*_{n,t}N^*_{n,t} = C^*_{n,t} - (1-\delta)\, C^*_{n,t-1} + \frac{\omega_n}{2}\left(\pi^*_{n,t} - 1\right)^2 \tag{3.42}$$

Finally, market clearing in the tradable sector implies:

$$\begin{aligned} A^*_{f,t}N^*_{f,t} &= (1-\alpha)\, C_{T,t}\left[\alpha S_t^{\eta-1} + (1-\alpha)\right]^{\frac{\eta}{1-\eta}} \\ &\quad +\alpha C^*_{T,t}\left[S_t^{\eta-1}(1-\alpha) + \alpha\right]^{\frac{\eta}{1-\eta}} + \frac{\omega}{2}\left(\pi^*_{f,t} - 1\right)^2 \end{aligned} \tag{3.43}$$

Budget constraints and current account

If monopolistic firms are owned by the inhabitants of the country in which they are located, the resources-expenditure balance of the borrower is given by the budget constraint (3.24), holding with equality around a deterministic steady state. Therefore, by substituting for real profits and for (3.40) and (3.39) in the borrower's budget constraint, the resource constraint of the representative agent in country H is:

$$\begin{aligned} &(1-\alpha)\frac{C_{T,t}S_t^{1-\eta}}{\alpha + (1-\alpha)\, S_t^{1-\eta}} - (1-\alpha)\, C^*_{T,t}\frac{\left[1-\alpha+\alpha S_t^{1-\eta}\right]^{\frac{\eta}{1-\eta}}}{\left[\alpha + (1-\alpha)\, S_t^{1-\eta}\right]^{\frac{1}{1-\eta}}} \\ =\ & b_t - R_{t-1}\frac{b_{t-1}}{\pi_{T,t}} \end{aligned} \tag{3.44}$$

Equation (3.44) shows that the inflows of foreign resources net of interest payments (the RHS) needs to equalize the consumption of tradables at Home, net of Foreign consumption of tradables (weighted for terms of trade). Equation (3.44) is also the current account equation for country H. More explicitly, we define the current account of country H (in real terms of home tradable consumption) as the variation of home-currency assets (in real terms of tradable consumption)[40], i.e.:

[40] In our numerical simulations, we will focus on the ratio of the current account over GDP.

$$
\begin{aligned}
ca_t &= \left(\frac{b_{t-1}}{\pi_{T,t}} - b_t\right) && (3.45)\\
ca_t &= x_{,t}A_{n,t}N_{n,t} + A_{h,t}N_{h,t}\frac{P_{h,t}}{P_{T,t}} - C_{T,t} - x_t\left(C_{n,t} - (1-\delta)\,C_{n,t-1}\right) && (3.46)\\
&\quad -\frac{b_{t-1}}{\pi_{T,t}}\left(R_{t-1}-1\right) - \frac{1}{P_{T,t}}\left[P_{n,t}\frac{\omega}{2}\left(\pi_{n,t}-1\right)^2 + P_{h,t}\frac{\omega}{2}\left(\pi_{h,t}-1\right)^2\right]
\end{aligned}
$$

Clearly, by substituting $A_{n,t}N_{n,t}$ and $A_{h,t}N_{h,t}$ with (3.39) and (3.40) and equating (3.45) and (3.46) we obtain equation (3.44).

In the rest of the world, the corresponding resource constraint is:

$$
\begin{aligned}
& C^*_{T,t}\frac{S_t^{\eta-1}(1-\alpha)}{\left[S_t^{\eta-1}(1-\alpha)+\alpha\right]} - R^*_{t-1}\frac{b^*_{t-1}}{\pi^*_{T,t-1}} && (3.47)\\
&= -b_t + (1-\alpha)\,C_{T,t}\frac{\left[\alpha S_t^{\eta-1} + (1-\alpha)\right]^{\frac{\eta}{1-\eta}}}{\left[(1-\alpha)\,S_t^{\eta-1}+\alpha\right]^{\frac{1}{1-\eta}}}
\end{aligned}
$$

Equation (3.47) can be also interpreted as a current account equation of country F.

3.2.7 Monetary policy

The recent house prices boom and the prospect of a global downturn as a consequence of sharp softening in housing sectors have triggered a debate on wether policy makers should respond to house prices. Conventional mainstreams agree that central bankers should respond to asset price changes only when they affect inflation, output and expectations (Mishkin, 2007). However, there are "benefits to be derived from leaning against the wind...(and)..increas(e) interest rates to stem the growth of house price bubbles and help restrain the building of financial imbalances" (IMF, 2008).[41]

There is also a general agreement on the desiderability to target inflation only in the sectors

[41] For more discussion on house prices and monetary policy targets see Borio and White (2004), Bordo and Jeanne (2002).

where prices are stickiest.[42] In this light, by letting prices free to move in the flexible-prices sectors, the monetary authority avoids output swings in the sticky-price sectors to stabilize the overall price index.

While refraining from normative issues, we limit our analysis to the effects of stochastic shocks in presence of alternative policy stances. In our framework, we assume that exchange rates are completely flexible and policy makers do not engage in any specific exchange rate policy. This leaves the central bank three possible targets: durable goods inflation, tradable goods inflation and/or domestically-produced tradables' inflation. In the following, when focusing on the response of the economy to shocks, we will track the adjustment dynamics with different Taylor rules.

In our simplified exercise, we assume that the policy maker does not aim to stabilize output. Clearly, as remarked by Iacoviello (2005) in an analogous framework, output targeting may be a source of possible policy trade-offs. For the moment we suppose that each policy maker react both to durables' and home-produced tradables' inflation according to the following Taylor rules:

$$\frac{R_t}{\bar{R}} = \left(\frac{\pi_{h,t}}{\bar{\pi}_h}\right)^{\Phi_{1,h}} \left(\frac{\pi_{n,t}}{\bar{\pi}_n}\right)^{\Phi_{2,h}} \tag{3.48}$$

$$\frac{R_t^*}{\bar{R}^*} = \left(\frac{\pi_{f,t}^*}{\bar{\pi}_f^*}\right)^{\Phi_{1,f}} \left(\frac{\pi_{n,t}^*}{\bar{\pi}_n^*}\right)^{\Phi_{2,f}} \tag{3.49}$$

In a two country setup, nominal determinacy requires Φ_1 and/or Φ_2 to be sufficiently large; we assume that the monetary policy is set so as to ensure sufficient conditions for nominal determinacy (see Benigno and Benigno, 2000). Notice finally that the above simple rules are not efficient: any change affecting the natural interest rates will likely entail an inflationary/deflationary bias.

[42] See Aoki (2001) and Benigno (2004).

3.2.8 Equilibrium conditions

For each monetary policy in country H and F, the equilibrium of our world economy is defined by (3.13) and (3.14) holding with equality around the deterministic steady state (see discussion in the following section), (3.15), (3.16),(3.25), (3.24), (3.26), (3.17), (3.27) and the no-Ponzi game condition, (3.23)[43]. In the tradable production sector, (3.31) and (3.33) need to hold while in the durables production sector (3.35) and (3.36). Market clearing is insured by (3.38)-(3.44) and (3.19). Finally purchasing parity conditions need to hold.

3.3 Steady state

In this section we focus on the qualitative features and the dynamic properties of the steady state. Once we have proved the existence of a "well behaving equilibrium", it is possible compute it analytically (see the Appendix).

3.3.1 Dynamic properties of the steady state

In order to show the existence of a determinate steady state and on its features we first shift our attention to the (modified) steady-state Euler equations of our model.

Consider first equations (3.26) and (3.59) –see the Appendix – at the steady state. Notice also that in steady state, $\pi_T = \nu \pi_T^*$ – indeed terms of trade are fixed in steady state (where ν is the steady-state depreciation rate of the nominal exchange rate). Also, long-run values of tradable inflation coincide with the target of the monetary policy for tradables in the Home and in the Foreign country, respectively. We can thus pin down the long-run value of ψ, i.e.:

$$\psi = 1 - \frac{\beta}{\mu} \tag{3.50}$$

implying that

$$1 > \psi > 0 \tag{3.51}$$

whenever $0 < \frac{\beta}{\mu} < 1$. Notice however that since $1 > \mu > \beta > 0$, inequality (3.51) always

[43] Given the collateral constraint, the no-Ponzi game condition is not binding.

holds. Eventually, Assumption 2 reduces to the Becker (1980) and Becker and Foias (1987, 1994) condition (see the following Proposition).

Proposition 1 *Under assumptions 1-3 and a monetary policy that insures nominal determinacy, if the system is sufficiently close to the deterministic steady state, then* $b_t = (1-\chi)\, C_{n,t} x_t$ *for every* $t \geq 0$.

Proof. The formal proof is in Becker (1980) and Becker and Foias (1987, 1994) with zero-borrowing constraints. In order to ensure the existence of a "dominant consumer" in this model, we need to focus on (modified) Euler equations. Given that the saver is a consumption smoother, the ratio of her/his steady-state marginal utilities is equal to one, i.e.:

$$\frac{U^*_{T,t}}{U^*_{T,t+1}} = R^*_t \frac{\mu}{\pi^*_{T,t+1}} = 1 \tag{3.52}$$

at the contrary, equation (3.17) shows that whenever at the steady state $0 < \psi < 1$,

$$0 < 1 - \beta \frac{R_t}{\pi_{T,t+1}} \frac{U_{T,t+1}}{U_{T,t}}$$

and the borrower is thus the "dominated consumer". Indeed, Assumption 2 ensures that $0 < \psi < 1$. ■■

Proposition 1 implies that in our framework the borrower is always debt-constrained. Indeed, given that the Lagrangian multiplier ψ is positive, the constraint must be binding.[44]

In addition, Proposition 1 states that there exists an unique steady state, which is characterized by a non-zero level of external liabilities. The steady state is also dynamically determinate. Indeed, as in the standard model of Becker (1980), the steady state is determined by the Euler equation of the patient agent and does not depend on initial conditions. It follows that the steady state of our system is not characterized by unit roots. Proposition 1 also allows thus to introduce the following corollary:

[44] See also Iacoviello (2005) for additional discussion.

Corollary 2 *If Proposition 1 holds, the dynamics of our two-country economy with heterogeneous agents and imperfect markets are not characterized by unit roots.*

The above Corollary needs some comments. Indeed, the pioneer analysis of Obstfeld and Rogoff (1995) shows that when markets are incomplete, the steady state of an open economy is generally subject to unit roots. This means that the steady state depends on initial values (i.e., initial external assets/liabilities) and transient shocks have long-run effects.

In a two-country framework, Cole and Obstfeld (1991) rule out unit roots under particular functional forms for utility. Analogously, Corsetti and Pesenti (2001) introduce an unitary elasticity of substitution between domestic and foreign goods.[45] This specification provides full risk sharing, renders securities redundant and implies a zero current-account balance for every period, also in presence of home bias. In turn, a zero current-account balance refrains transitory shocks from having long-run effects. Alternatively, the non-stationarity of the steady state is ruled out by Cavallo and Ghironi (2002); in their model, zero steady-state liabilities are an endogenous result. More recently, in a framework of overlapping generations and heterogeneous discounting, Ghironi et al. (2005) extend this result to the case of non-zero long-run external liabilities; in this setting, the possibility for the (relatively more) patient agent to hold all capital in steady state is ruled out by the absence of intergenerational bequest. This result should not surprise the reader; empirical evidence (see Lane and Milesi-Ferretti, 2001, 2002) show that non-zero long-run external assets/liabilities are a common phenomenon.

Schmitt-Grohe and Uribe (2003) provide a detailed analysis on stationarity-inducing methods to rule out unit roots in a small-economy framework.[46] The proposed modifications to the standard model aim at inducing stationarity of the equilibrium dynamics: they make the steady-state independent from initial conditions. However, when stationarity is induced by portfolio adjustment costs or interest rate premia, long-run assets are determined exogenously. Indeed there is an exogenous level of debt around which the adjustment function is centered.[47]

[45]For a literature review see Lane (2001).

[46]Bodestein (2006) provides an analogous analysis applied to a two-country world.

[47]Punzi (2007) and Callegari (2007) incorporate collateral constraints and discount rates heterogeneity in a two country business-cycle model. In their framework, the steady-state value of external debt is eventually pinned down by standard portfolio adjustment costs.

When stationarity is induced by endogenous discounting, the discount factor function is centered around an exogenous steady-state level for consumption.

In Becker (1980) seminal article, the steady state is endogenously defined by the Euler equation of the "dominant consumer" (i.e., the patient agent) and it is independent from initial conditions. By extending Becker (1980) seminal result to the case of a two-country framework, we also rule out unit roots. Indeed, in steady state, the Euler equation only depends on the parameters of the model. This result implies that (sufficiently small) stochastic shocks do not have long-run effects and the steady state is not subject to unit roots dynamics.[48]

Notice finally that these results do not depend neither on nominal rigidities neither on the introduction of durables.

3.4 Current account dynamics and housing market spillovers

The recent swings in house prices and their dramatic consequences have triggered a debate on the implications for the global economy. Indeed, the current degree of financial globalization has likely enhanced the international transmission of shocks. In a recent article on the sustainability of US current account and the dollar, Krugman (2007) discusses the linkage between the value of the dollar and the housing sector.[49] He provides a stylized analysis on the effects of a fall of both housing prices and the value of the dollar; finally, he evaluates the implications of the monetary policy stance.

In our analysis we are not interested in evaluating the sustainability of current trends. Instead, we incorporate Krugman's (2007) message so as to explore the transmission mechanisms of shocks arising from the housing market. Indeed, as mentioned in the Introduction, the linkage between international capital flows, exchange rate and the wealth of collateral-constrained agents has proved a fundamental transmission mechanism in the context of currency crises in

[48] The steady state is globally unique. Having said that, the dynamic properties of the steady state hold only around the steady state itself. The reason is that the dynamic properties cannot but be calculated by inspecting the Jacobian matrix (and thus, after linearization). Same considerations apply for the stability properties of the model (see Becker and Foias 1997)

[49] His framework is based on an analysis of the Tobin q. He argues that US Tobin q is mainly driven by the housing sector. Since home returns and foreign returns are linked through the exchange rates, developments affecting housing, also affect exchange rate dynamics: an increase in the Tobin q is thus associated to a dollar appreciation.

emerging countries. While extending the focus to the developments of housing wealth, we extend the application of the open-economy "Bernanke-Gertler" effect to the analysis of current account dynamics.

In this section, we study the impact of developments affecting the housing market and their implications for the dynamics of international variables. We will first analyze the effect of likely stochastic shocks affecting consumption patterns. As in Ferrero et al.(2008), preference shocks are here meant to capture structural factors entailing changes in consumption patterns.[50]

Finally, in order to proxy the recent financial liberalization, we introduce a (very) persistent stochastic shock affecting the tightness of the collateral constraint and we see how this affect current account and exchange rate dynamics.

3.4.1 Calibration

In order to have a quantitative insight of the dynamics of the model, we proceed by simulating the response of our economy to stochastic shocks. Our parametrization is consistent with the recent literature and is based in particular on Monacelli (2007, 2008), Obstfeld and Rogoff (1995, 2000, 2004) and Faia and Monacelli (2007).

Quarterly discount factors are set respectively to $\mu = 0.99$ and $\beta = 0.98$.[51] The structure of the model implies that the real interest rate of the F country is pinned down by the discount factor of the dominant consumer, and thus is equal to $\frac{1}{\mu}$.

We stick on Monacelli (2007) benchmark value for houses depreciation rate and we set $\delta = (0.025)^{1/4}$, which is consistent an annual depreciation between 1,5% and 3%. Analogously, we adopt the average loan-to-value ratio on home mortgages for the period 1952-2005 in U.S. and let $\chi = 0.25$.[52]

We calibrate γ by assuming that the share of durable spending on total spending in Home, $\left(\frac{\delta C_n}{\delta C_n + C_T}\right)$, is equal to 0.2. This is consistent with U.S. data on spending. We calibrate v by assuming that the steady state level of labor in F is one third of one unit of time, so as to proxy European trends; the inverse elasticity of labor supply is assumed to be equal to 3.

[50] Iacoviello (2005) introduces preference shocks so as to proxy temporary tax advantages or sudden increases in demand for houses fuelled by optimistic consumer expectations – the latter interpretation reflects our logic. In the same vein, Krugman (2007) considers a shock in investors' expectations.

[51] Caroll and Samwick (1997) estimate discount factors to be in a range between 0.91 and 0.99.

[52] It is also consistent with current trends in industrial countries, see IMF (2008).

The elasticity of substitution amongst single varieties (for each sector in each country) is set equal to 8. This implies a mark-up of about 15%. The elasticity of substitution between the basket of home and foreign goods is reasonably assumed to be lower than 8 and is set equal to 2. Moreover, the share of Home (Foreign) good consumption in the Home (Foreign) country, α, is set equal to 0.7.

In our benchmark parametrization, we follow the standard literature on sticky prices adjustment and we assume a frequency of four quarters for tradable goods.[53]

3.4.2 Debt constraints

In presence of agents' heterogeneity in the degree of impatience, debt constraints are a key element to ensure the uniqueness and the dynamic determinacy of the steady state. We assume that debt contracts are stipulated in nominal terms, so as to proxy reality in developed countries. Monacelli (2007, 2008) and Iacoviello (2005), show that shocks entailing an increase in inflation imply a flow of wealth from the lender to the borrower. To the contrary, shocks entailing a decrease in inflation imply a beneficial wealth transfer in favour of the lender. In our two-country world, the international transmission of wealth effects is due to terms of trade. Indeed, everything else fixed, an improvement of terms of trade has positive income effects and can enhance consumption.

If debt contracts are in nominal terms, real interest payments depend on aggregate tradable inflation in domestic currency – and thus, on terms of trade also. Clearly, if real interest rates increase, borrowers suffer higher interest payments. Terms of trade affect both Home-produced tradables inflation and aggregate-tradable inflation (CPI inflation) depending on nature of the shock. In the following, we will study the role of terms of trade in the international transmission of wealth effects.

3.4.3 Demand shocks

We now focus the attention on the possible effects of two different demand shocks: i) a positive shock in preferences for housing in the debtor country, H (so as to proxy the demand shock for

[53] See Bils and Klenow (2004) and Monacelli (2007) for some recent discussion on the frequency of price adjustment in U.S.

houses in countries characterized by great amount of net external liabilities); a positive shock on preferences for tradables in country F (so as to proxy a generalized inflation boom). Indeed, as suggested by Iacoviello and Neri (2008), demand shocks have played a major role in explaining the cyclical volatility of houses and prices in recent years.

Demand shock in the housing sector

Suppose now that country H is affected by a positive shock in consumers' preferences such that its inhabitants desire to buy more houses. For simplicity, we suppose that the shock has a log-normal distribution such that[54]:

$$\begin{aligned} p_{n,t} &= \rho_n p_{n,t-1} + u_t \\ u_t &\sim (iid) \end{aligned}$$

where we let $\rho_n = 0.85$, following the calibration of Iacoviello (2005).

We see that in response to an increase in households' appetite for real properties, the financial accelerator is at work. The increase in the stock of real assets – and thus, in housing relative prices – entails a better access to credit.[55] Since agents in country H are impatient, they use all their collateral to obtain credit. In turn, this allows them to consume tradables and to further increase their stock of houses: external debt increases on impact and continues to gradually accumulate before the shock is absorbed. The accumulation of collateral is accommodated by a gradual decrease in the user cost of durables. By inspecting equation (3.18) – see also the Appendix –, one can see that the user cost of durables is a positive function of both the interest rate and the relative price of durables.[56] Both relative prices and interest rates jump on impact and decrease gradually.

Given the expansion of debt, interest rates increase; notice in particular that in absence of nominal rigidities, the interest rate and the level of inflation are determined by the Taylor rule

[54] The utility function is thus: $\max E_0 \left\{ \sum_{t=0} \beta^t U_t \left(e^{p_{n,t}} C_{n,t}, C_{T,t}, N_t \right) \right\}$

[55] Indeed, the marginal value of borrowing, ψ, decreases.

[56] For a focus on the role of ψ, see the above discussion. See also Monacelli (2008) with a similar (but not identical) collateral constraint.

together with the modified Euler equations of the borrower. Indeed, the central bank reacts to the expansionary effects of the shocks by tightening interest rates.

Notice in particular that in our two-country world, developments affecting housing prices are transmitted internationally through terms of trades. The transmission channel is evident in absence of prices rigidities. If prices are flexible, real marginal costs need to be constant in each period. In our case, given an identical mark-up for all sectors[57] and mobile labor across sectors, it always needs to hold:

$$\frac{1}{x_t}\frac{-U_{N,t}}{U_{T,t}A_{n,t}} = \frac{(\varepsilon-1)}{\varepsilon} = \frac{1}{A_{h,t}}\frac{-U_{N,t}}{U_{T,t}}\left[\alpha+(1-\alpha)\,S_t^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{3.53}$$

implying that the increase in housing relative prices entails an improvement in terms of trade. Eventually this will transmit the effect of the shock to country F.[58]

Borrowers enjoy thus a positive income effect. As you can see in (3.53), the decrease in S enhances the consumption of tradables. Moreover, the decrease in the relative price of foreign goods prompts borrowers to substitute Home with Foreign tradables consumption; this switching effect entails a trade deficit. The trade deficit together with a (quantitatively small) increase in interest payments triggers the insurgence of a current account deficit.

Thanks to the income effect, aggregate labor supply does not increase on impact in country H (as expected, it does increase in country F): indeed, the positive income effect deriving from the improvement in terms of trade offsets the increase in aggregate consumption in H. Aggregate labor gradually increases as soon as terms of trade start deteriorating, before of the absorption of the shock.

Notice in particular that we have assumed that debt contracts are stipulated in nominal terms. In a closed economy framework, Iacoviello (2005) and Monacelli (2007) show that an increase in inflation implies a shift of resources in favour of the borrower. In our framework, the impact of shocks on inflation is affected by terms of trade. In turn, the impact of terms of trade on the inflation index of tradables is generally not uniquely signed. Indeed, terms of trade affect both the Home-produced tradables inflation[59] and the CPI index of inflation, according

[57] Clearly, qualitative results would hold also with different mark-up.
[58] For a more detailed discussion on the trasmission of shocks, see the following section.
[59] Marginal costs in the Home tradable sector are a positive functon of terms of trade

to definition (3.1) and (3.20).[60] Clearly, if CPI inflation rises, real interest payments decrease.

The fall in terms of trade plays a double role in response to the shock: on the one hand, by making Foreign consumption cheaper, it accommodates borrowers' impatience and willingness to increase current consumption. On the other hand, it enhances lenders' accumulation of Foreign currency assets and thus, their future investment incomes – we remind the readers that lenders are consumption smoother. Notice indeed that the amount of Foreign external assets is a negative function of terms of trade, according to (3.21). This is due to a revaluation effect of lenders' assets. Notice that the structure of our two-country world is characterized by the fact that borrowers do not lend; therefore, borrowers' net external liabilities coincide with their gross external liabilities. In turn, given that borrowers' debt is denominated in Home currency, assets revaluation effects associated to exchange rate swings are ruled out.[61] Conversely, given that lenders are not indebted and their holdings of gross and net external assets coincide, their wealth is subject to revaluation effects associated to exchange-rate fluctuations. As you can see in Figure 3.2, the fall in S has a positive impact on Foreign external assets, which is associated to the initial appreciation of the nominal exchange rate. Notice also that given home bias, the improvement of terms of trade is associated to a real appreciation in the first period; indeed, the (nominal and real) exchange rate undershoots on impact so as to depreciate during the adjustment process. At the end of the adjustment, the nominal exchange rate is eventually more depreciated than its pre-shock value.

The current account deficit is eventually absorbed as a consequence of the gradual decrease in borrowers' tradable consumption on the one hand; and the decrease in borrowers' ability to access to credit, on the other.

To conclude, the above results show that in response to a preference shock for durables, the increase in house relative prices is transmitted internationally through an improvement in terms of trade: house prices and terms of trade move in opposite directions. Borrowers experience a current account deficit; the exchange rate appreciates on impact and depreciates throughout the adjustment. Notice interestingly that these trends are consistent with US data

[60] In response to the preference shock for housing $\pi_T < \pi_h$, if prices are flexible. However, if prices of tradable goods are sticky, the opposite is true and $\pi_T > \pi_h$.

[61] In presence of Home-denominated debt and foreign denominated assets, revaluation effects on foreign assets can have a significant impact on the dynamics of external debt. See, among many others, Gourinchas and Rey (2007), Obstfeld and Rogoff (2004), IMF (2005a).

on current account, house prices and exchange rates during the last years. Around 2005, the US have experienced a steeper increase in house prices, a temporary undershooting of the exchange rate and a deterioration of the current account (see Figure 3.1a, 3.1b).[62] Our results are consistent with Iacoviello and Neri (2008). According to their empirical analysis, during the 2000-2005 period, housing preference shocks played a major role in explaining the boom in housing investments and prices in US.

Nominal rigidities In Figure 3.3, we show the effect of the above shock in presence of different types of price rigidities. We compare the dynamics of variables when tradables have a 4 quarter frequency in price adjustment (case 1 in Figure 3.3), with the case of price flexibility (case 2) and the case where only house prices have a 4 quarter frequency in price adjustment (case 3).

Figure 3.3 shows that the scenario characterized by price flexibility is intermediate between the other two. The main effect of price rigidities lies in their impact on the relative price durables-tradables.

If tradables are sticky, the relative price of durables jumps higher than when prices are flexible because the price of tradables does not immediately adjust to the expansionary effect of the shock. Notice that since the prices of durables are here kept flexible, it still needs to hold: $\frac{1}{x_t}\frac{-U_{N,t}}{U_{T,t}A_{n,t}} = \frac{(\varepsilon-1)}{\varepsilon}$. In the tradable-goods sector inflation is now pinned down by the New Keynesian Phillips curve, (3.31). Therefore, terms of trade and the relative price of durables are no more directly linked; still, they continue to move inversely. Following the jump in x, terms of trade improve but in a smaller extent (indeed, the price of Home produced goods increase more slowly).

Following the stronger jump in relative prices, the user cost of durables jumps higher; thus, agents afford a smaller amount of real properties. Having said that, the increase in the value of houses allows them to continue borrowing so as to increase consumption; there is thus a switching effect in favour of tradable consumption. Notice however that the dynamics of the current account are not affected by price rigidities; indeed, the switching effect in favour of tradable consumption is offset by the smaller extent of the switching effect in favour of Foreign

[62] Clearly, to drive any conclusion more empirical work would be needed.

tradables (due to the weaker improvement of terms of trade). Finally the effects of price rigidities are not quantitatively relevant for the dynamics of the real exchange rate[63]. As in Ferrero et al. (2008), the behavior of real-international variables do not differ significantly from the flexible-case scenario: they depend mainly on real factors.

Monetary policy We now focus on the role of monetary policy. We choose to assume the perspective of the Home central bank; we aim at investigating the effects of different targets on the dynamics of our collateral-constrained open economy. Given the structure of our two-country two-sector economy, policy makers in each country can choose alternative targets: tradable goods inflation, Home-produced goods inflation and durable-goods inflation.

We consider the following alternative simple monetary rules:

$$\frac{R_t}{\bar{R}} = \left(\frac{\pi_{h,t}}{\bar{\pi}_h}\right)^{\Phi_{1,h}}, \Phi_{1,h} \to \infty \tag{3.54}$$

$$\frac{R_t}{\bar{R}} = \left(\frac{\pi_{T,t}}{\bar{\pi}_T}\right)^{\Phi_{1,h}} \left(\frac{\pi_{n,t}}{\bar{\pi}_n}\right)^{\Phi_{2,h}} \tag{3.55}$$

$$\frac{R_t}{\bar{R}} = \left(\frac{\pi_{T,t}}{\bar{\pi}_T}\right)^{\Phi_{1,h}}, \Phi_{1,h} \to \infty \tag{3.56}$$

together with the benchmark rule, (3.48). According to rule (3.54) (i.e., scenario 2 in the simulations of Figure 3.4), the monetary authority stabilizes inflation of Home produced goods only (i.e., $\Phi_{2,h} = 0$ and $\Phi_{1,h}$ is large). According to rule (3.55), scenario 3, the central bank targets both CPI-inflation and inflation in the housing sector. Notice however that this specification implies that the monetary authority directly targets also the inflation of imported goods; in this way, s/he directly responds to shocks that may be imported from abroad.[64] Finally, when the central bank follows rule (3.56), scenario 4, s/he responds to tradable (CPI) inflation but disregards the trends of house prices.

In Figure 3.4, we show the adjustment dynamics following a preference shock for housing under the likely scenario where the prices of tradables are sticky in both countries.

[63]However, price rigidities in the tradable sector introduce larger swings in the nominal exchange rate.

[64]One could think of commodities such as oil. For instance, by targetting core inflation, the Fed does not directly respond to the increase in oil prices.

When the policy maker implements rule (3.56) aggregate tradable inflation is best stabilized, together with terms of trade. Therefore, most of the weight of the adjustment of the shock is carried by the relative price of houses, x (on impact it jumps higher). This implies that, on impact, agents afford a smaller amount of durables; having said that, they can afford more tradable consumption – thanks to a stronger wealth effect related to the increase of value of their assets and a switching effect in favour of tradable consumption. Notice however that the impact of the shock on the current account is not significantly different from the one in the other scenarios. Indeed, given the smaller improvement in terms of trade, the stronger switching effect in favour of tradable consumption is offset by a weaker switching effect in favour of Foreign consumption.

Notice finally that when the central bank targets housing prices, inflation is generally less stabilized in all baskets of goods.[65] Given that we have assumed flexible house prices, this result confirms the desirability of targeting stickiest prices only. The monetary policy is indeed less effective in stabilizing the effects of preference shocks even if interest rates rise higher. In addition, as in Iacoviello (2005) and Monacelli (2007), targeting house prices does not significantly improve the adjustment of real (international) variables; the current account (in real terms of tradable consumption) is not significantly affected by the introduction of house prices in the Taylor rule. Moreover, when the central bank targets also housing inflation, the nominal and real exchange rate experience larger swings.

Demand shock for tradables in country F

The two-country structure of our model prompts us to analyze the transmission of shocks from one country to the other. In this section we focus on the transmission of a positive demand shock from country F to H.

Suppose that country F is affected by positive demand shock for tradables. For simplicity, we suppose that the shock has a log-normal distribution such that:

[65] Moreover, when the central bank implements rule (3.54), aggregate tradable inflation is more volatile but the single baskets of durable inflation and Home tradables are better stabilized. This implies that the relative price of houses is (slightly) better stabilized. The nominal interest rate increases less, leading in turn to smaller swings in the nominal exchange rate.

$$\begin{aligned} p_{T,t} &= \rho_T p_{T,t-1} + u_t \\ u_t &\sim (iid) \end{aligned}$$

where we let $\rho_T = 0.85$, as above.[66]

Focusing first on F, we see that, as expected, the positive shock triggers a strong increase in tradable consumption.

If prices are flexible, real marginal costs in each sector need to be equal to the mark up. This needs to hold in each period. We have assumed for simplicity the same mark-up for all sectors[67], therefore:

$$\frac{1}{x_t^*} \frac{-U_{N,t}^*}{e^{p_{T,t}} U_{T,t}^* A_{n,t}^*} = \frac{(\varepsilon - 1)}{\varepsilon} = \left[\alpha + (1-\alpha) S_t^{\eta-1}\right]^{\frac{1}{1-\eta}} \frac{-U_{N,t}^*}{A_{f,t}^* U_{T,t}^* e^{p_{T,t}}}$$

In a closed economy, the term $\left[\alpha + (1-\alpha) S_t^{\eta-1}\right]^{\frac{1}{1-\eta}}$ would be equal to 1 and the relative price x^* would be fixed.[68]. In our framework, both x^* and S represent a wedge and are allowed to accommodate the shock by moving proportionally.

Figure 3.5 shows that the preference shock for tradables makes relative prices, x^*decrease; this entails a proportional fall in S.[69] Notice that the decrease of both variables leaves consumption and labor less scope for jumping – dampening thus the expansionary effect of the shock on tradable consumption. Eventually, a decrease in x^* also dampens the substitution effect between durables and non-durables following the shock in preferences. Indeed, given that lenders are not collateral constrained, all shocks affecting the real price of durables can entail a strong substitution between goods – see the arbitrage equation, (3.25).

The variation in terms of trade transmits the shock to country H. Focusing now on country H, we see that the variation of S implies a variation of x in the opposite direction with respect

[66] Lenders' utility function is thus: $\max E_0 \left\{\sum_{t=0} \beta^t U_t \left(C_{n,t}^*, e^{p_{T,t}} C_{T,t}^*, N_t^*\right)\right\}$

[67] If mark up are not identical, results wouldn't qualitatively change.

[68] See Monacelli (2007).

[69] In country F, the Taylor rule triggers a jump in interest rates. This makes current consumption more expensive and gives the lenders an incentive to accumulate more assets (partly offsetting the effect of the shock). The accumulation of assets is in turn enhanced by the revaluation effects associated to the change in S.

to x^*.

$$\frac{1}{x_t}\frac{-U_{N,t}}{U_{T,t}A_{n,t}} = \frac{(\varepsilon-1)}{\varepsilon} = \frac{1}{A_{h,t}}\frac{-U_{N,t}}{U_{T,t}}\left[\alpha+(1-\alpha)\,S_t^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

The decrease in S entails a positive income effect for borrowers and triggers an increase in tradable consumption, C_T . Eventually, the shock is also transmitted to the relative price, x, which moves opposite to S. This adds a positive wealth effect on borrowers' collateral. Borrowers can thus obtain more credit and further increase their level of consumption. Surprisingly, the effect of the preference shock in country F has thus an expansionary effect on borrowers' consumption. Thanks to the strong income effect, tradable consumption in country H (slightly) increases more than aggregate labor in the tradable sector.[70] However, country H experiences a temporary (small) trade surplus – price effect. The Home central bank reacts thus promptly to keep this expansionary effect under control; then, interest payments together with the increase in terms of trade entail a (small) current account deficit which is slowly absorbed.[71]

If prices are flexible, the dynamics of interest rates and inflation can be pinned down by combining the modified Euler equations with the Taylor rules. Given that the simple monetary rules used by policy makers are not efficient, inflation is allowed to jump. The nominal exchange rate accommodates the stance of both policies and the real exchange rate co-moves with terms of trade: it (slightly) appreciates on impact and depreciates during the adjustment. In steady state, the nominal exchange rate is more depreciated with respect to its the pre-shock level.

The shock on preferences makes lenders increase their tradable consumption on impact but deteriorates their terms of trade – contributing to a (small) current account deficit for country F. Then, the increase in interest rate payments, together with the gradual increase in terms of trade entails a current account deficit for country H. The current account deficit is eventually balanced when the effect of the variation in interest incomes and the increase of country H tradable consumption are absorbed.

[70] Indeed, the decrease in terms of trade has a negative income effect for lenders; this prompts them to supply more labor.

[71] Even if tradable consumption decreases more than labor in the tradable sector. Clearly, the quantitative impact of the shock on both $U_{T,t}$ (and thus, C_T) and $U_{N,t}$ (and thus N) depends on the parameters of the utility function.

Nominal rigidities In order to investigate the transmission of the shock in presence of nominal price rigidities we consider the following scenarios: flexible prices (case 1 in Figure 3.6a, 3.6b); nominal rigidities for Home tradables only (case 2); nominal rigidities for Foreign tradables only (case 3); nominal rigidities for both Home and Foreign tradables (case 4).

Figure 3.6a 3.6b shows that at a qualitative level, the impact of the shock in country F depends on the existence of nominal rigidities in country F: the dynamics of case 1 (3) are indeed analogous to those of case 2 (4) but the impact of rigidities is quantitatively very small. If prices are rigid in country F, the dynamics of the relative price of durables, x^* and of terms of trade, S are still linked. However, agents expectations and sectorial inflation create a wedge between the dynamics of the two variables – following the New Keynesian Phillips curve, (3.33). In the tradable sector, π_f^* cannot jump and most of the pressure of the expansion falls on the non-tradable sector. As a consequence, relative prices, x^* experience larger fluctuations – and terms of trade decrease less. There is a quantitatively small substitution effect in favour of durables when the relative price of houses is lower.

As expected, the effect of the shock on the relative price of durables in country H depends on price rigidities in country H (see Figure 3.6b).[72] The shock will be eventually transmitted on tradable and durable consumption through the relative price of durables, x: the higher the relative price, the smaller the amount of purchased houses.

We notice that the introduction of nominal rigidities has a very small quantitative effect on current account and exchange rate dynamics.

Monetary policy We now focus on the role of the monetary policy. We continue assuming the perspective of the Home policy maker and we analyze the effects of a demand shock for tradables when the policy maker implements rules (3.48), (3.54)-(3.56), respectively. Notice however that given our focus on shocks affecting country F, we allow the central bank in country F to modify her/his benchmark Taylor rule.[73]

Figure 3.7 shows the response of variables to a preference shock for tradables in country

[72] In country H, the dynamics of case 1 (2) are indeed analogous to those of case 3 (4) and the impact of rigidities is quantitatively very small.

[73] In particular, whenever country H does not target house prices (this correspond to case 2 and 4 in the numerical simulations), the central bank in country F follows the simple benchmark rule:

F, when the central bank follows different monetary rules. Notice that the above transmission mechanisms apply also here; however, changing the monetary rule has quantitatively smaller implications. Rule (3.48) and (3.55) have very similar implications. When instead rule (3.56) and (3.57) are implemented, terms of trade are better stabilized and large part of the adjustment is carried by a jump in the relative price of houses, x. This entails a stronger increase in tradable consumption and a stronger fall in the consumption of durables. This effect is offset by a smaller switching effect in favour of Foreign consumption. Smaller amounts of interest payments entail a smaller current account deficit during the adjustment. The current account follows the line of these trends when the central bank implements rules (3.54) and (3.57).

Notice finally that when the policy maker targets house prices also, interest rates react more strongly, entailing larger nominal exchange rate swings (this is consistent with the above results[74]). In turn, higher interest rates entail larger interest payments and thus, a (slightly) larger current account deficit during the adjustment.

3.4.4 Financial liberalization

In this Section we focus on the effects of financial liberalization. We aim at proxying the liberalization process that characterized housing finance during the last decades.[75]

In our simplified framework, we proxy the process of financial liberalization as a loosening of collateral requirements. In particular, we introduce a stochastic (very persistent) shock hitting the share of real properties that can be used as a collateral, $(1-\chi)$. Increasing this share allows agents to access larger amounts of credit. Following the same lines of the previous sections, we assume that the shock of financial liberalization has a log normal distribution such that:

$$\frac{R_t^*}{\bar{R}^*} = \left(\frac{\pi_{f,t}^*}{\bar{\pi}_f^*}\right)^{\Phi_{1,f}}, \quad \Phi_{1,f} \to \infty \tag{3.57}$$

Notice however that trends in H would not be significantly affected if country F continued using the benchmark rule (3.48).

[74] Analogously, targeting house price also is not efficient: on inmpact, inflation jumps higher in all baskets – except for house prices.

[75] The liberalization of both international financial markets and housing finance took different forms and followed diverse patters in different coutries. IMF (2008) provides an index proxying the degree of liberalization of mortgages markets in a panel of countries.

$$l_t = \rho_l l_{t-1} + u_t$$
$$u_t \sim (iid)$$

where we let $\rho_T = 0.99$. In Figure 3.8a we show the adjustment dynamics when benchmark Taylor rules are implemented and prices of tradables are rigid. If the collateral constraint is loosened, the marginal value of borrowing, ψ decreases. To the contrary, the marginal utility of durable consumption increase together with their relative price, x[76]; thus, the user cost of durables increase so that agents afford smaller amounts of houses and substitute durable with tradable consumption.[77] The positive wealth effect on borrowers' assets adds on the loosening of the collateral constraint; therefore, borrowers can further increase their level of debt (remember that borrowers are not consumption smoother). In turn, this boosts consumption.

The above transmission channels imply that the increase in house relative prices translates into an improvement in terms of trade[78]. Borrowers enjoy thus also a positive income effect that further enhances their level of (tradable) consumption.[79] Country H experiences the insurgence of a trade deficit that is at the roots of a current account deficit. Moreover, interest payments increase and add on the deterioration of the current account. Indeed, the reaction of the central bank to the expansionary effect of the shock triggers an increase in interest rates in country H. The exchange rate undershoots on impact and depreciates during the adjustment. In steady state, the nominal exchange rate is more depreciated with respect to the initial situation. Notice finally that the dynamics of the current account, exchange rates and house prices are qualitatively similar to the dynamics following a preference shock for tradables. Having said that, the current account deficit implied by a preference shock for tradables is quantitatively more significant (comparison not shown in the simulations).

[76] Notice however that prices increase both in the tradable and in the nontradable sector.

[77] See the above discussion on the impact of ψ on the user cost of durables; see also the Appendix.

[78] In response to the shock, if all prices are flexible, $\pi_T < \pi_h$; if tradables prices are sticky, terms of trade dampen tradable inflation on impact (i.e., $\pi_T < \pi_h$) but enhance inflation during the rest of the adjustment (i.e., $\pi_T > \pi_h$) – dampening in turn real interest rates (in terms of tradable consumption).

[79] The shock has a negative impact on aggregate labor in country H and a positive effect on aggregate labor in country F.

As in the previous section, price rigidities do not have an important effect on international real variables[80]. Indeed, the current account deficit is not significantly affected by price rigidities; as in the previous section, price rigidities entail a larger jump in the relative price of houses, associated with a smaller improvement in terms of trade. Therefore, the stronger switching effect in favour of tradable consumption is offset by the weaker switching effect in favour of foreign tradables (see Figure 3.8b).

Finally, in Figure 3.8c we show the impact of different simple monetary rules; as you can notice, the effects follow the same logic of the previous sections; as for the preference shock for houses, the current account is not significantly affected by the monetary policy stance (but the nominal and the real exchange rate are).

3.5 Quantitative insights and dynamics of the model

In this Section we will continue exploring the dynamics characterizing our model. We will first analyze the effect of an aggregate technology shock in country F and its transmission to country H. We will then focus on an aggregate shock in country H so as to compare the adjustment dynamics of the two countries.

3.5.1 Aggregate technology shock in country F

Suppose that country F is affected by a positive aggregate technology shock; the shock we consider hits both housing and the tradable production sector so that productivity coefficients A_f^*, A_n^* increase in the same way. As in the previous sections, we suppose that the productivity shock has a lognormal distribution, such that:

$$\begin{aligned} a_{f,t} &= \rho_a a_{f,t-1} + u_t \\ u_t &\sim (iid) \end{aligned}$$

[80] As expected, price rigidities entail relatively larger swings in the nominal exchange rate.

and we suppose for simplicity $A^*_{f,t} = A^*_{n,t} = e^{a_{f,t}}$.[81]

In response to a positive technology shock in country F, lenders increase savings (i.e., net external assets), consumption[82] and work less in the tradable sector.[83]

At the same time, the decrease in marginal costs makes Foreign goods cheaper; therefore, S decreases[84]. When prices are flexible, $\frac{1}{x^*_t}\frac{-U^*_{N,t}}{U^*_{T,t}A^*_{n,t}} = \left[\alpha + (1-\alpha)\, S_t^{\eta-1}\right]^{\frac{1}{1-\eta}} \frac{-U^*_{N,t}}{A^*_{f,t}U^*_{T,t}}$; thus, house relative prices decrease.

The fall in terms of trade has a negative effect on savers' income (and a positive effect on their net external assets) and makes Home consumption more expensive. Terms of trade, labor supply and consumption show an hump-shaped trend. This depends on the following offsetting effects: i) the direct effects of the shock on the variables (entailing an increase in F-tradable consumption, a decrease in S and and in F-labor supply) and ii) the feedbacks associated the negative income effect implied by the fall of terms of trade in the first period (See also equation (3.43))

The shock is thus transmitted to country H through a positive income effect for borrowers; tradable production in country H decreases but tradable consumption is sustained by imports. Since $\frac{1}{x_t}\frac{-U_{N,t}}{U_{T,t}A_{n,t}} = \frac{1}{A_{h,t}}\frac{-U_{N,t}}{U_{T,t}}\left[\alpha + (1-\alpha)\, S_t^{1-\eta}\right]^{\frac{1}{1-\eta}}$, the effect of terms of trade is transmitted to relative house prices[85] and enhances borrowers' collateral. The financial accelerator is thus at work. The hump-shaped trend of consumption follows both the trends of terms of trade and of house relative prices (the latter, through the financial accelerator channel).[86]

In country F, the productivity shock triggers a fall in inflation; there is thus scope for a monetary loosening. Notice also that the shock in F entails a (smaller) fall in inflation also in country H; indeed, marginal costs in H are a negative function of relative house prices and a positive function of terms of trade. Thus, the effects associated to the changes in relative prices

[81] We assume that $\rho_a = 0.85$. Engel and Wang (2008) estimate the coefficient related to a productivity shock in the non-durable sector only to be equal to 0.87 and the one in the durable sector as 0.9.

[82] Moreover, the associated loosening of the monetary policy prompts agents to consume more – see the following.

[83] Clearly, quantitative results depend on the paramenters of the model.

[84] The effect is eventually enhanced by an appreciation of country-H nominal exchange rate.

[85] Notice that house prices do not increase; housing relative prices increase because the price of the aggregate basket of tradables decreases more.

[86] Terms of trade trigger the transmission of the shock. The transmision channel depends on a positive net effect on borrowers' income. Indeed, aggregate consumption increase in country H and labor swings are quantitatively insignificant.

more than offsets the quantitatively (very) small increase in aggregate labor (see Figure 3.9c). Interest rates in country H are eventually determined by combining the Taylor rule with the Euler equations.

Focusing now on the international transmission of the shock, notice that the improvement of terms of trade triggers a trade deficit for country H (see Figure 3.9c). On impact, country H also experiences a current account deficit. Having said that, the decrease in interest payments eventually allows country H to carry a current account surplus throughout the adjustment. To the purpose of accommodating the monetary stance, the nominal exchange rate appreciates on impact and depreciates throughout the adjustment. Same considerations apply for the real exchange rate. In the long run, the nominal exchange rate is more depreciated than in the pre-shock situation.

The positive technology shock in country F entails thus an improvement of borrowers' terms of trade and an increase in house prices. During the whole adjustment process, borrowers experience a trade deficit. However, while on impact they also experience a (quantitatively not significant) current account deficit, decreasing interest payments help sustaining a current account surplus throughout the adjustment (see Figure 3.9c).

Finally, in Figure 3.9a and 3.9b we show the effect of nominal rigidities for tradables on the transmission of the shock. Price rigidities do not seem to significantly affect the international transmission of technology shocks. As in the above analysis, price rigidities introduce a wedge on relative prices entailing quantitatively small consumption switching effects.[87]

3.5.2 Aggregate technology shock in country H

Suppose now that country H is affected by a positive aggregate technology shock; as in the previous section, the shock we consider hits both the housing and the tradable production sector so that productivity terms A_h, A_n increase in the same way. As in the previous sections, we suppose that the shock has a lognormal distribution, such that:

[87] If central banks do not target house prices, the current account initially falls stronger. Moreover, during the adjusment country H experiences a smaller current account surplus due to a smaller fall in interest rates and interest payments (see Figure 3.9d).

$$
\begin{aligned}
a_{h,t} &= \rho_a a_{h,t-1} + u_t \\
u_t &\sim (iid)
\end{aligned}
$$

and we suppose for simplicity $A_{h,t} = A_{n,t} = e^{a_{h,t}}$.[88] In response to a positive productivity shock, borrowers increase their level of consumption and decrease their labor effort. In Figure 3.10a we compare the response of output and labor when either country H or F are subject to a positive productivity shock. Given that borrowers do not save and are impatient, the effect on consumption and on labor effort is stronger in country H; for the same reason, aggregate output in H increases in a smaller extent.

In response to a productivity shock in H, domestic goods become cheaper than Foreign ones. This entails an increase in terms of trade and borrowers experience thus a negative income effect. This effect is transmitted to house prices; if prices are flexible, $\frac{1}{x_t}\frac{-U_{N,t}}{U_{T,t}A_{n,t}} = \frac{1}{A_{h,t}}\frac{-U_{N,t}}{U_{T,t}}\left[\alpha + (1-\alpha)S_t^{1-\eta}\right]^{\frac{1}{1-\eta}}$. Therefore, the increase in terms of trade entails also a negative wealth effect on borrowers' real assets. These two effects partly offset the impact of the productivity shock on labor. Thus, labor effort show an hump-shaped response (see Figure 3.10a) and terms of trade respond in turn following an analogous trend – see equation (3.40). The decrease in houses relative prices makes houses less expensive and allow a better access to real properties.[89] The access to foreign credit allows borrowers to increase consumption above output (see Figure 3.10b) so that interest payments accumulate. There is thus a strong upward pressure on prices due to the increase in terms of trade and the decrease of house relative prices; for this reason (together with the strong upward pressure on consumption) inflation increase in all sectors.[90] Indeed, the downward pressure on prices due to the shock is more than offset by these effects.[91] The central bank needs thus to (slightly) tighten interest rates.

On impact, country H experiences a trade deficit that is at the root of a current account

[88] As above, we assume that $\rho_a = 0.85$

[89] Having said that, the marginal value of borrowing, ψ, decreases together with increasing external borrowing.

[90] Both P_h and P_n increase in country H. However, since terms of trade entail a stronger increase in P_T, house relative prices fall. Notice that the increase of Home currency foreign prices is mostly due to the depreciation of the nominal exchange rate.

[91] Notice that, everything else fixed, terms of trade keep aggregate tradable inflation higher ($\pi_T > \pi_h$). This keeps interest payments lower and partly offsets the negative income effects.

deficit; during the adjustment, the trade deficit translates into a trade surplus (at the roots of a current account surplus). There is a positive correlation between terms of trade and the real exchange rate; the (real and nominal) exchange rate depreciates on impact and makes Home goods cheaper; it appreciates during the adjustment. At the end of the adjustment, the nominal exchange rate is appreciated with respect to the initial level.

Finally, as expected, price rigidities do not affect the current account. While affecting the relative price of houses, they induce a small switching effect in favour of tradable consumption; however, since terms of trades rise relatively higher, there is a switching effect in favour of Home-produced tradables (see Figure 3.10c).[92]

3.6 Concluding remarks

We have focused on the current account dynamics of a two-country world populated by heterogeneous agents in their degree of impatience. We have shown that if the inhabitants of country H are more impatient than the ones of the Foreign country, and their willingness to consume is limited by a collateral constraint, we can extend Becker (1980) and Becker and Foias (1987) seminal result to an open-economy dimension. Indeed, given that the H-inhabitants are not consumption smoother, they always prefer to borrow as much as possible and the collateral constraint is binding in each period. In the long run, the collateral-constrained country is characterized by a positive amount of external debt and a balanced current account; non-zero liabilities are thus an endogenous result of our model and are consistent with a dynamically determinate steady state.

We have then analyzed the effect of shocks and their international transmission. While considering the developments of housing wealth, we have in practice extended the application of Krugman (1999) open-economy "Bernanke-Gertler" effect to the analysis of current account dynamics. The dynamics of our model have shown the transmission channel that links developments affecting the housing sector to current account and exchange rate dynamics. House

[92] The effect of different monetary stances are analogous. Having said that, when the monetary policy implements rule (3.56), the nominal interest rate jumps less, entailing a smaller nominal depreciation. Terms of trade are better stabilized and the consumption of Foreign tradables is thus enhanced. However, this effect on trade is offset by a switching effect in favour of durable consumption; the latter is triggered by a decrease in relative house prices.

prices and terms of trade are linked; therefore, all shocks affecting house prices are transmitted internationally. Conversely, all shocks entailing a change in terms of trade affect the housing sector. Interestingly, the co-movements that we have individuated in response to likely shocks – emphasized by Iacoviello and Neri (2008) – are in line with US recent trends.

We have also studied the role of price rigidities. As in Ferrero et al. (2008), price rigidities do not significantly affect the trends of real international variables: while entailing larger swings in the nominal interest rates, they don't significantly affect the current account.

Finally, we have focused on the monetary policy stance. Our study extends the results of Iacoviello (2005) and Monacelli (2007) in an open-economy framework; indeed, targeting house prices does not improve the adjustment dynamics of (international) real variables such as the current account. However, consistently with Ferrero et al. (2008) our results show that targeting flexible prices (i.e., house prices) can induce larger swings in nominal interest rates and in the nominal exchange rate.

3.7 Appendix

3.7.1 The detailed optimization program of the borrower.

Utility function:

$$\max E_0 \left\{ \sum_{t=0} \beta^t U\left(C_t, N_t\right) \right\}$$

Complete budget constraint, in nominal terms:

$$P_{T,t}C_{T,t} + P_{n,t}\left(C_{n,t} - (1-\delta)\, C_{n,t-1}\right) + R_{t-1}D_{t-1} - q_t R^*_{t-1} D^*_{t-1} \leq D_t - q_t D^*_t + W_t N_t + \sum \Gamma$$

where D are the bonds issued at Home in Home currency and D^* are bonds issued in the Foreign country in Foreign currency. The individual budget constraint in real terms of tradable consumption is:

$$C_{T,t}+x_t\left(C_{n,t}-(1-\delta)\,C_{n,t-1}\right)+R_{t-1}\frac{d_{t-1}}{\pi_{T,t}}-q_t R^*_{t-1}d^*_{t-1}\frac{P^*_{T,t-1}}{P_{T,t}}\leq d_t-q_t d^*_t\frac{P^*_{T,t}}{P_{T,t}}+\frac{W_t N_t}{P_{T,t}}+\sum\frac{\Gamma}{P_{T,t}} \tag{3.58}$$

Using price index definitions, and the law of one price, (3.58) can be rewritten as:

The collateral constraint is:

$$d_t-q_t d^*_t\frac{P^*_T}{P_{T,t}}\leq(1-\chi)\,C_{n,t}x_t$$

First order conditions:

a) Arbitrage leisure/consumption:

$$U_{T,t}=\lambda_t$$

b) Arbitrage tradable consumption/durable services:

$$x_t U_{T,t}=U_{n,t}+\beta\,(1-\delta)\,E_t\left\{U_{T,t+1}x_{t+1}\right\}+U_{T,t}\psi_t\,(1-\chi)\,x_t$$

c) Modified Euler equation

$$1=E_t\left\{\frac{U_{T,t}}{U_{T,t+1}}\frac{\pi_{T,t+1}}{R_t}\right\}\frac{(1-\psi_t)}{\beta}$$

b) Optimal condition for foreign securities:

$$1=E_t\left\{\frac{U_{T,t}}{U_{T,t+1}}\frac{q_t}{q_{t+1}}\frac{\pi_{T,t+1}}{R^*_t}\right\}\frac{(1-\psi_t)}{\beta} \tag{3.59}$$

Equations (3.58) and (3.59) implies that the following non-arbitrage condition needs to hold, i.e.:

$$R_t=E_t\left\{\frac{q_{t+1}}{q_t}\right\}R^*_t$$

3.7.2 The user cost of durables during the transition

We now aim at disentangling the impact of each component of the RHS of (3.18) on the user cost of durables, during the transition toward the steady state. We thus log linearize equation (3.18) so as to solve for the log deviations of variables. Let the log deviation of the user cost be denominated as $\hat{Z}_t$. Then:

$$\hat{Z}_t = (x+\Omega)\frac{\hat{x}_t}{Z} + (A+\Omega)\frac{\hat{\psi}_t}{Z} + (\Gamma + A)\frac{E_t\{\hat{\pi}_{T,t+1}\}}{Z} + (A+\Gamma)\frac{E_t\{\hat{x}_{t+1}\}}{Z} - \frac{\hat{R}_t}{Z}(A+\Gamma)$$

where we define:

$$\begin{aligned} A &\equiv \psi(1-\delta)\frac{\pi_T x}{R} \\ \Omega &\equiv -\psi(1-\chi)x \\ \Gamma &\equiv -(1-\delta)\frac{\pi_T x}{R} \end{aligned}$$

Moreover, variables without time index are steady-state values and hatted variables are variables in log deviation from the deterministic steady state. We also notice that in steady state: $\psi = 1 - \frac{\beta}{\mu}$.

Results show that:

i) An increase in x has a positive effect on the user cost (as expected) as long as:

$$x[1-\psi(1-\chi)] > 0$$

which is comfortably satisfied with our benchmark parametrization.

ii) An in crease in both $E_t\{\hat{\pi}_{T,t+1}\}$ and $E_t\{\hat{x}_{t+1}\}$ has a negative effect on the user cost as long as

$$-x\frac{\pi_T}{R}(1-\delta)(1-\psi) < 0$$

which is comfortably satisfied with our benchmark parametrization (and for any reasonable ones).

iii) An increase in the nominal interest rate entail an increase in the user cost as long as

$$-x\frac{\pi_T}{R}(1-\delta)(\psi-1)>0$$

which is comfortably satisfied in our benchmark parametrization (and for any reasonable ones).

iv) An increase in ψ has a negative effect on the user cost as long as:

$$\psi x\left[(1-\delta)\frac{\pi_T}{R}-1+\chi\right]<0$$

which is satisfied with our calibration. This implies that an increase in the marginal value of borrowing makes agents substitute tradables in favor of houses. Relative prices and ψ have thus opposite effects on the user cost (for everything else fixed).

Notice however that for $\delta \to 0$, this result is reversed and we recover Monacelli (2008) result with a slightly different collateral constraint. Moreover, if we impose the following constraint:

$$b_t=(1-\delta)(1-\chi)E_t\left\{C_{n,t}x_{t+1}\frac{\pi_{T,t+1}}{R_t}\right\}$$

as in Monacelli (2008), the impact of an increase in ψ_t pushes up the user cost of durables as long as

$$x\psi(1-\delta)\left[\pi_T-(1-\chi)\pi_n\right]>0$$

which is comfortably satisfied with our benchmark parametrization. This has important implications for consumption patterns, see Monacelli (2008).

3.7.3 Steady state: analytical solution

We now explicitly calculate the steady state of our model. Long term inflation levels are defined by the target of the monetary policy (we assume that $\bar{\pi}_n^*=\bar{\pi}_n=\bar{\pi}_T^*=\bar{\pi}_T=\bar{\pi}_f^*=\bar{\pi}_h=1$) and the saver's discount rate pins thus down both the real rate of return in F, $RR=\frac{1}{\mu}$ and $\psi=1-\frac{\beta}{\mu}$, as in Monacelli (2007).

In steady state, the price rigidities à la Rotemberg are no more at stake; the steady state of our framework coincides with the flexible prices steady state. Marginal costs are thus equal to the mark up. Assuming for simplicity the same mark-up for all sector in both countries,

$$mc_n = mc_h = mc_f^* = mc_n^* = \frac{\varepsilon - 1}{\varepsilon} \tag{3.60}$$

Supposing for analytical simplicity that the elasticity of substitution between tradables and houses θ is unitary[93], the consumption aggregator assumes a Cobb-Douglas specification; conditions (3.60), (3.30) (3.32) (3.34), (3.25), (3.37) and (3.16) allow us to pin down the durable and non-durable level of consumption both at Home and in Foreign and relative prices x and x^*:

$$C_n = \frac{(1-\gamma)}{vN^{\varphi}} \frac{e_1}{a_1} \tag{3.61}$$

$$C_T = \frac{\varepsilon - 1}{\varepsilon} \frac{\gamma}{vN^{\varphi}} \left[\alpha + (1-\alpha) S^{1-\eta}\right]^{\frac{-1}{1-\eta}} \tag{3.62}$$

$$x = \left[\alpha + (1-\alpha) S^{1-\eta}\right]^{\frac{-1}{1-\eta}} \tag{3.63}$$

$$C_n^* = \frac{(1-\gamma)}{vN^{*\varphi}} \frac{e_1}{a_2} \tag{3.64}$$

$$x^* = \left[\alpha + (1-\alpha) S^{\eta-1}\right]^{\frac{-1}{1-\eta}} \tag{3.65}$$

$$C_T^* = e_1 \frac{\gamma}{vN^{*\varphi}} \left[\alpha + (1-\alpha) S^{\eta-1}\right]^{\frac{-1}{1-\eta}} \tag{3.66}$$

where $e_1 = \frac{\varepsilon-1}{\varepsilon}, a_1 = 1 - \beta(1-\delta) - \psi(1-\chi)$ and $a_2 = 1 - \mu(1-\delta)$. Notice that in steady state the amount of borrowers' real properties is a positive function of the marginal value of additional borrowing, ψ. Indeed, the greater the value of borrowing, the larger the amount of collateral agents are wiling to hold. Analogously, the smaller the inverse of the loan-to-value ratio, the larger the amount of steady-state durables. Given the higher service they provide as collateral, agents have a stronger incentive to hold them in the long run.

Substituting (3.61) and (3.63) in (3.14) we obtain the steady-state level for net external debt in Home:

$$b = (1-\chi) \frac{(1-\gamma)}{vN^{\varphi}} \frac{e_1}{a_1} \left[\alpha + (1-\alpha) S^{1-\eta}\right]^{\frac{-1}{1-\eta}} \tag{3.67}$$

[93] We will keep this simplification during the simulation of our model.

and the one in Foreign:

$$b^{*}=b\left(\frac{(1-\alpha)\,S^{1-\eta}+\alpha}{\alpha S^{1-\eta}+1-\alpha}\right)^{\frac{1}{1-\eta}}$$

We pin down steady-state levels for N_h, N_n and N by substituting (3.61) and (3.63) in (3.38),(3.39) and(3.40), i.e.:

$$\begin{aligned} N_h &= \alpha e_1\frac{\gamma}{vN^{\varphi}}\left[\alpha+(1-\alpha)\,S^{1-\eta}\right]^{-1} \\ &\quad +(1-\alpha)\,e_1\frac{\gamma}{vN^{*\varphi}}\frac{\left[(1-\alpha)+\alpha S^{1-\eta}\right]^{\frac{\eta}{1-\eta}}}{\left[\alpha+(1-\alpha)\,S^{\eta-1}\right]^{\frac{1}{1-\eta}}} \end{aligned} \tag{3.68}$$

$$N_n=\delta\frac{(1-\gamma)}{vN^{\varphi}}\frac{e_1}{a_1} \tag{3.69}$$

$$N=N_h+N_n \tag{3.70}$$

The terms of trade, S, are pinned down by substituting all above steady-state values in (3.44).

An analogous procedure allows us to find all Foreign steady-state values, i.e.:

$$N_n^{*}=\delta\frac{e_1}{a_2}\frac{(1-\gamma)}{vN^{*\varphi}} \tag{3.71}$$

$$\begin{aligned} N_f^{*} &= (1-\alpha)\,e_1\frac{\gamma}{vN^{\varphi}}\frac{\left(\alpha S_t^{\eta-1}+(1-\alpha)\right)^{\frac{\eta}{1-\eta}}}{\left(\alpha+(1-\alpha)\,S^{1-\eta}\right)^{\frac{1}{1-\eta}}} \\ &\quad +\alpha e_1\frac{\gamma}{vN^{*\varphi}}\left[\alpha+(1-\alpha)\,S^{\eta-1}\right]^{-1} \end{aligned} \tag{3.72}$$

3.7.4 Figures

Figure 3.1a. Trends in the nominal and effective CPI exchange rate (BIS database)

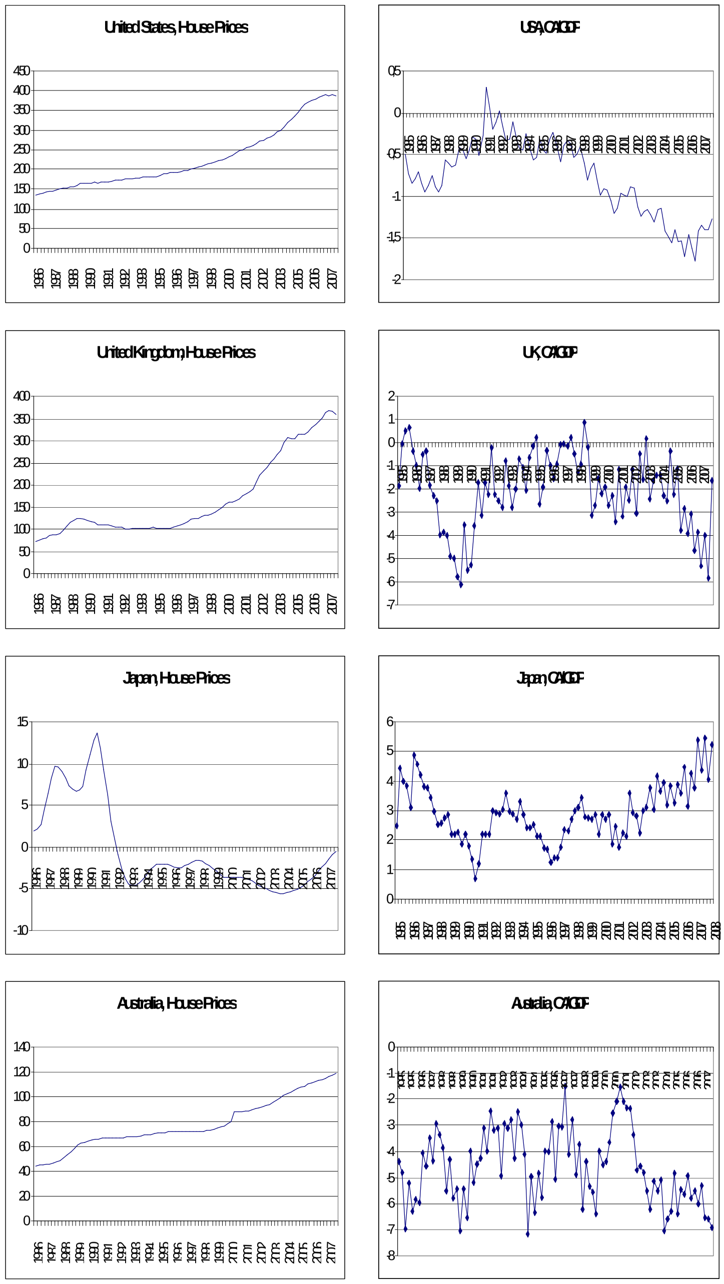


Figure 31b. House prices and current account

(Ecowin database)

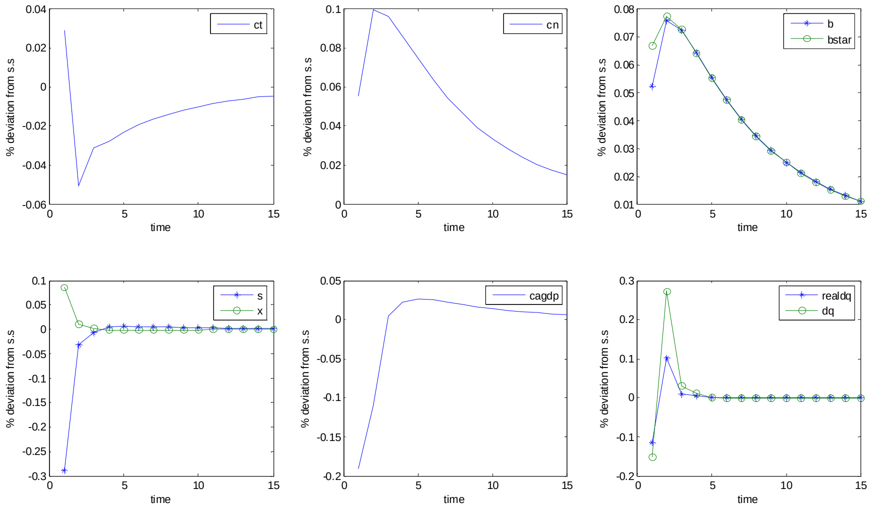


Figure 3.2. Preference shock for houses in H, flexible prices.

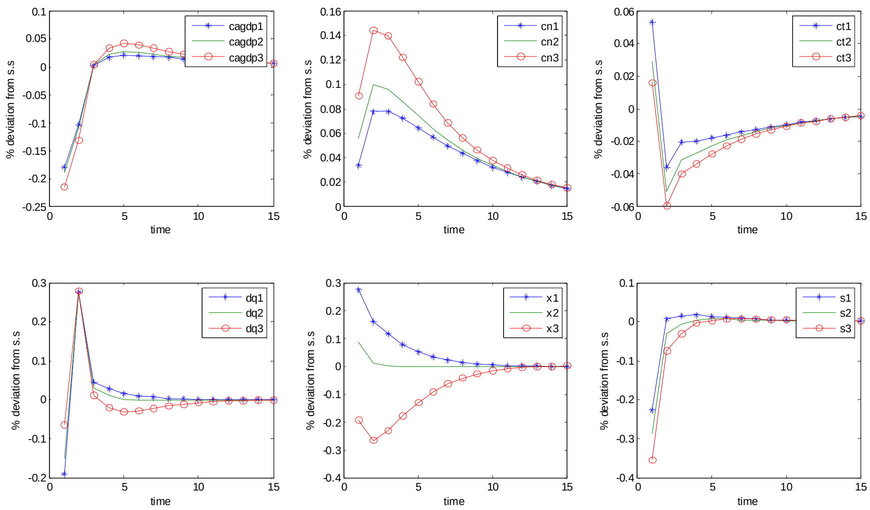


Figure 3.3. Preference shock for houses in H, different price rigidities.

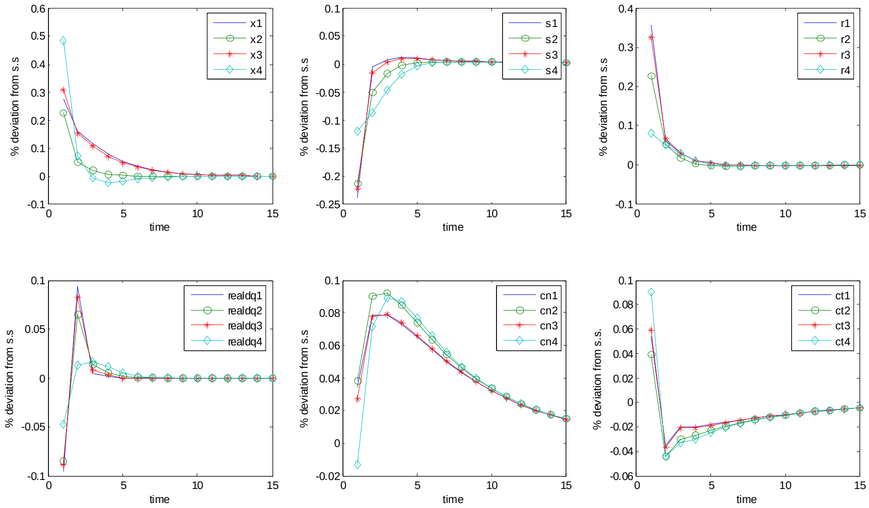


Figure 3.4. Preference shock for houses in H and Taylor rules.

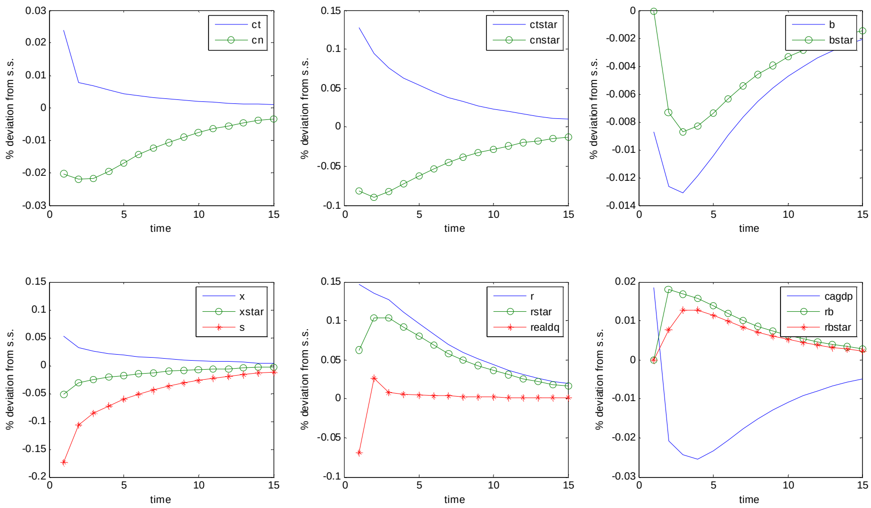


Figure 3.5. Preference shock for tradables in country F, flexible prices.

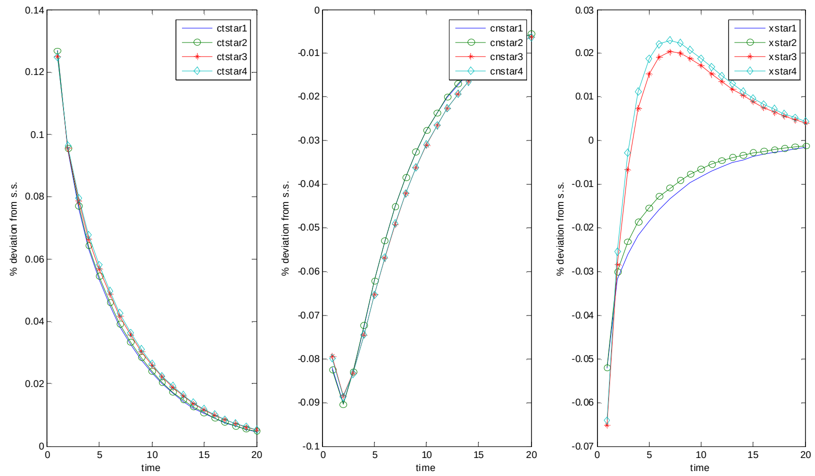


Figure 3.6a. Preference shock for tradables in country F,different price rigidities.

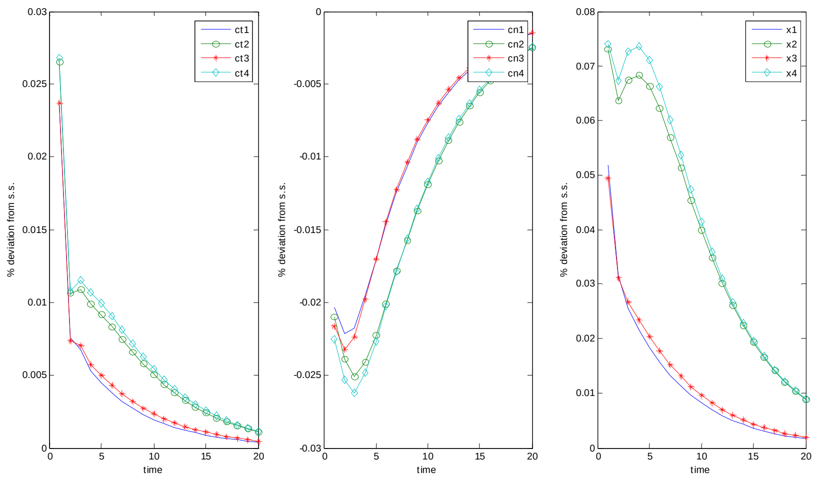


Figure 3.6b. Preference shock for tradables in country F,different price rigidities.

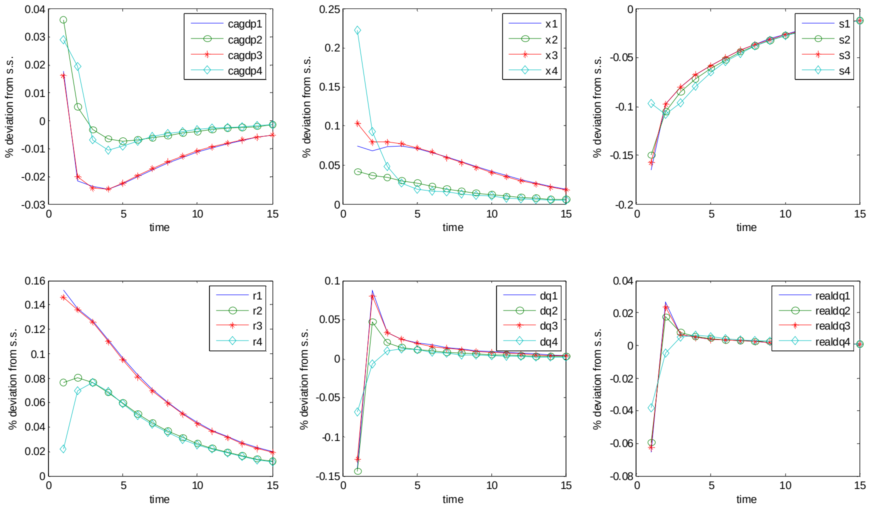


Figure 3.7. Preference shock for tradables in country F and Taylor rules.

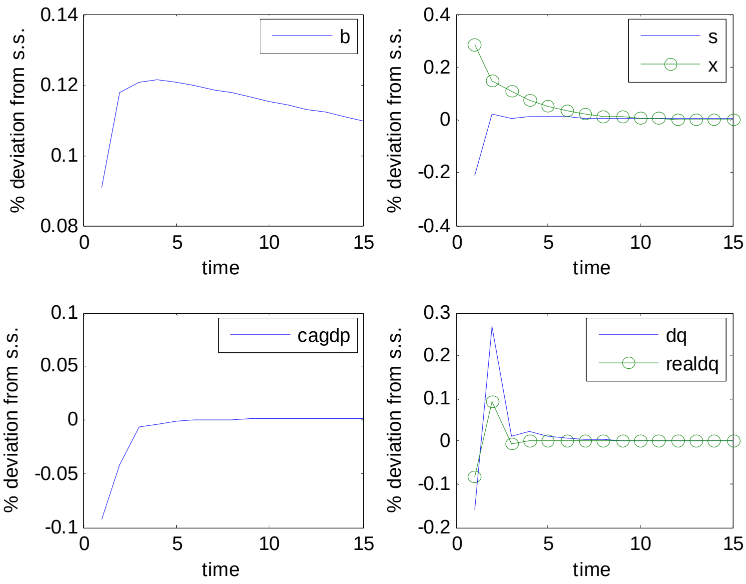


Figure 3.8a. Increase in financial liberalization.

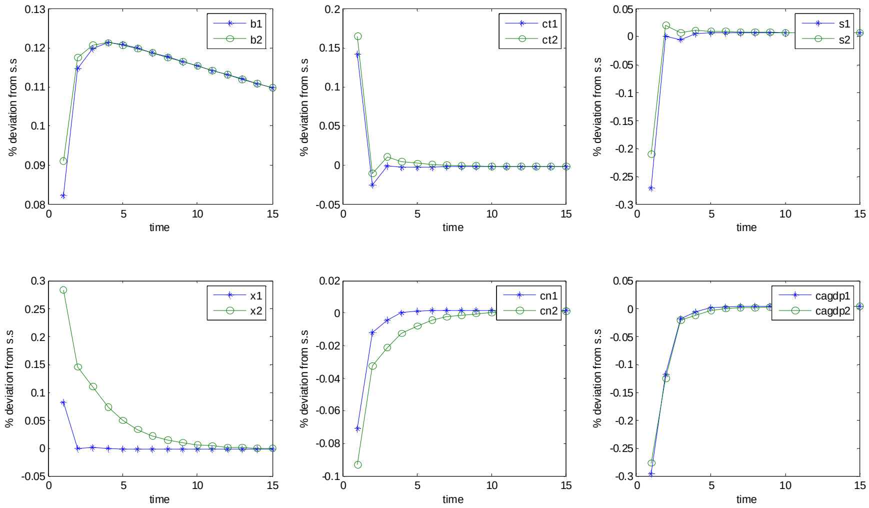


Figure 3.8b. Increase in financial liberalization, different price rigidities.

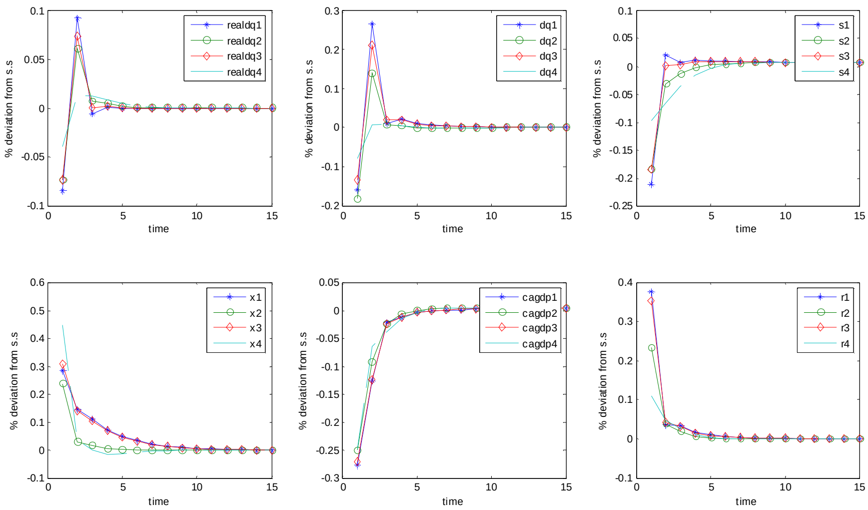


Figure 3.8c. Increase in financial liberalization, different Taylor rules.

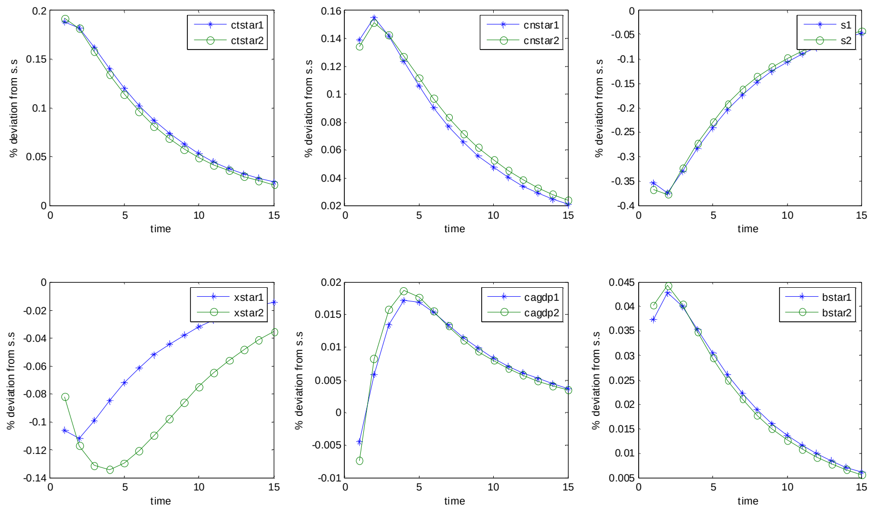


Figure 3.9a. Aggregate productivity shock in country F, price rigidities in country F.

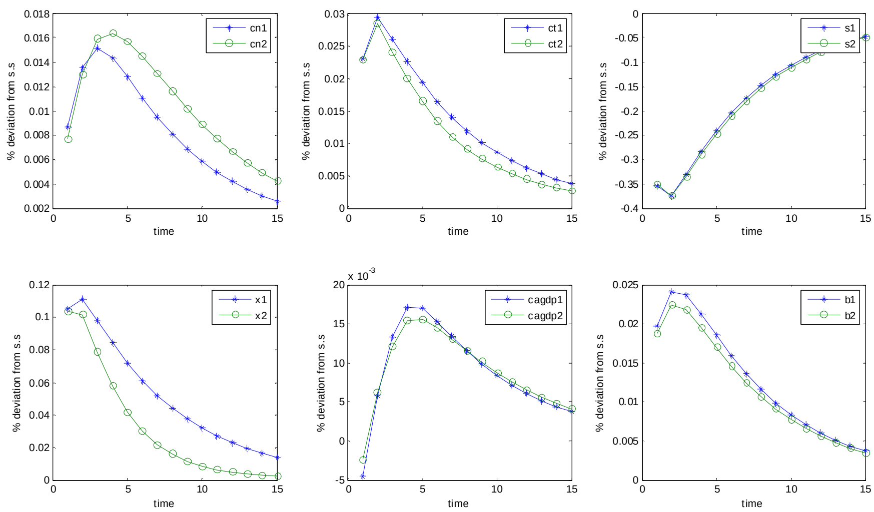


Figure 3.9b. Aggregate productivity shock in country F, price rigidities in country H.

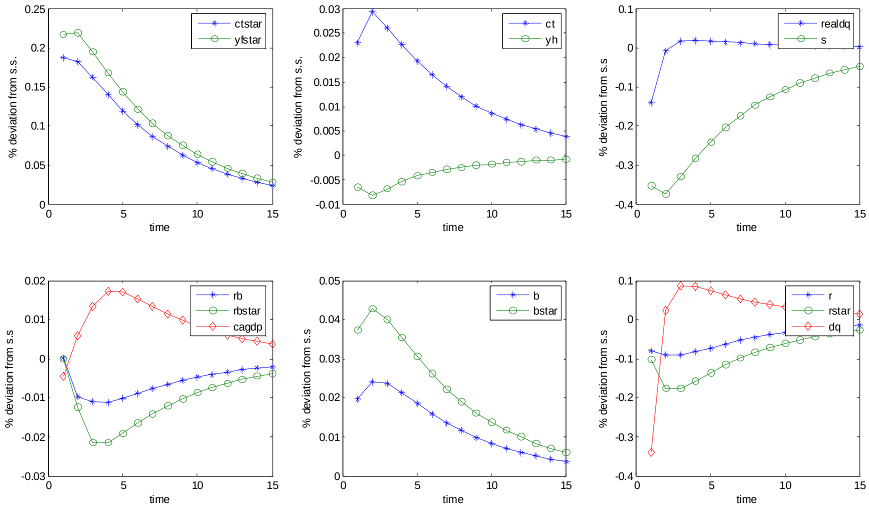


Figure 3.9c. International transmission of an aggregate technology shock in country F.

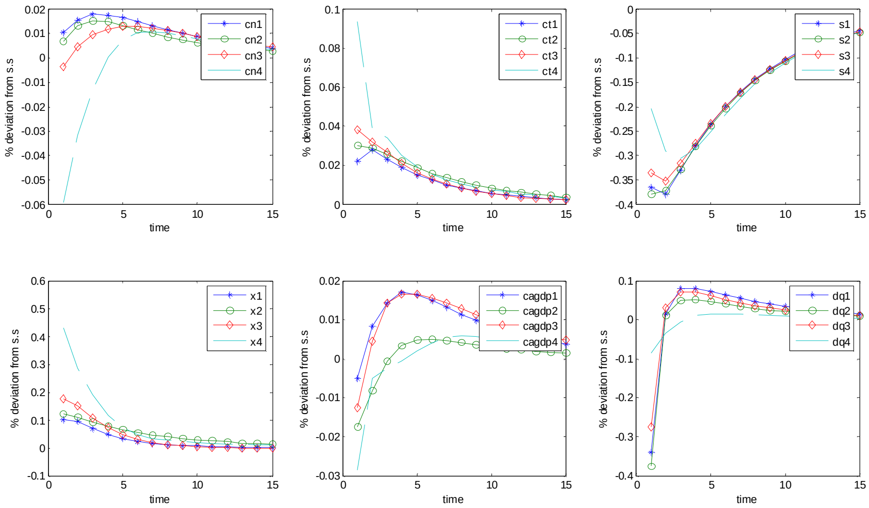


Figure 3.9d. International transmission of an aggregate technology shock in country F and Taylor rules.

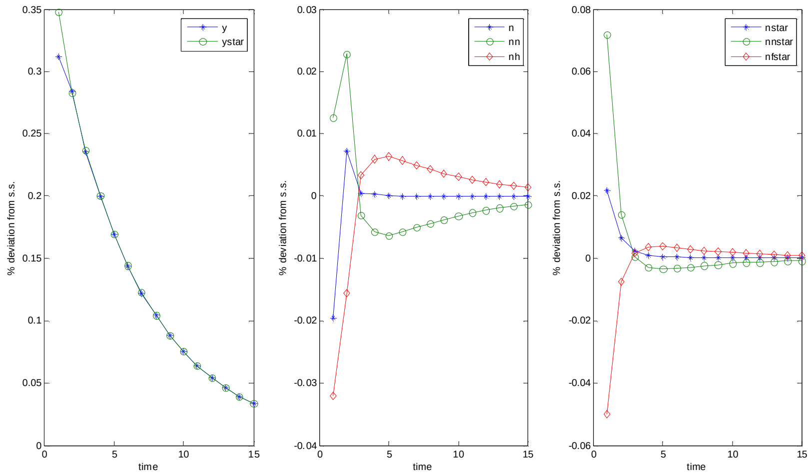


Figure 3.10a. Aggregate productivity shocks in country H and F, respectively.

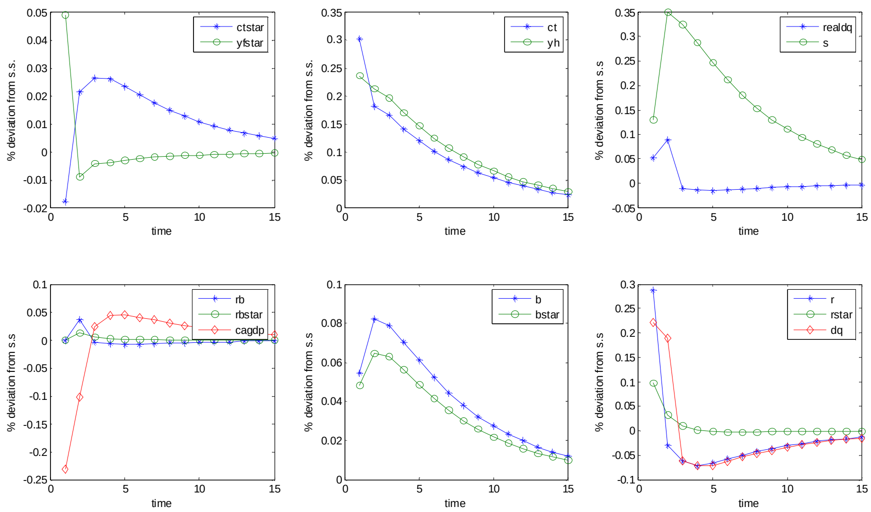


Figure 3.10b. Aggregate productivity shock in country H .

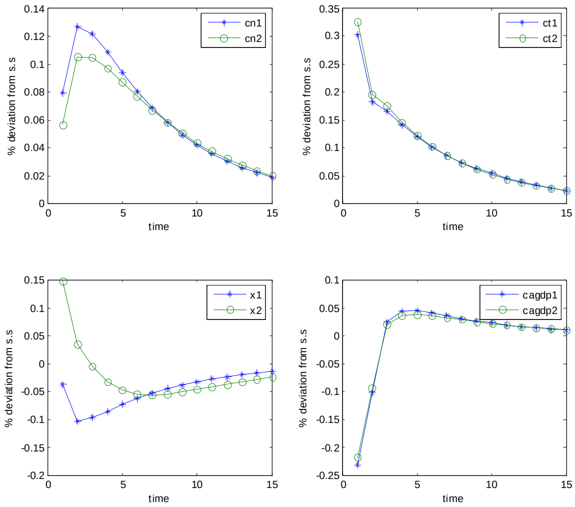


Figure 3.10c. Aggregate productivity shock in country H, price rigidities in country H.

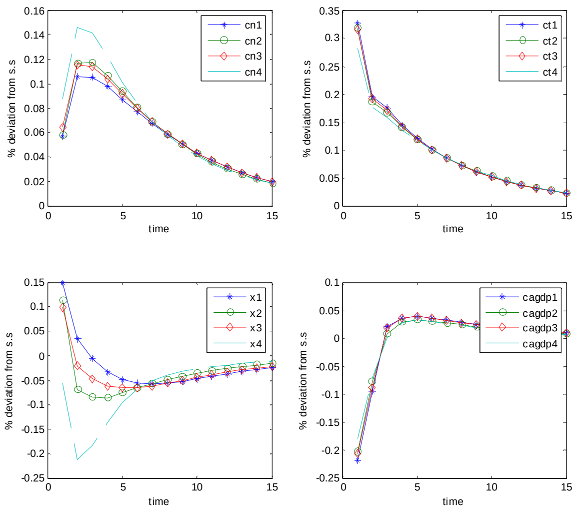


Figure 3.10d. Aggregate productivity shocks in country H and Taylor rules.

Notes on impulse response functions and calibration:

Numerical simulations represent the response of the economy following shocks of 1% standard deviation; time is in quarters. Impulse response functions represent percentage deviation from the steady state. dq refers to the depreciation rate of nominal exchange rate while $realdq$ to the depreciation rate of the real exchange rate. $cagdp$ refers to the ratio CA/GDP; rb represents real interest payments in the Home country and $rbstar$ represents interest incomes in country F. yh refers to the domestic production of Home tradables and $yfstar$ represents the Foreign production of Foreign tradables. All other symbols reflect the notation in the text.

Figure 3.2. Calibration: $\Phi_{1,h}$= $\Phi_{2,h}$ = $\Phi_{1,f}$ = $\Phi_{2,f}$ =1. All prices are here flexible.

Figure 3.3. Calibration: $\Phi_{1,h}$= $\Phi_{2,h}$ = $\Phi_{1,f}$ = $\Phi_{2,f}$ =1. Prices are flexible in country F. Variables indexed with 1 refer to the case of price rigidities in the tradable sector, in country H; variables indexed with 2 refer to the case of price flexibility in both sectors; variables indexed with 3 refer to the case of price rigidities in the housing sector, in country H.

Figure 3.4. Calibration: $\Phi_{1,f}$ = $\Phi_{2,f}$ =1 always. Nominal rigidities for tradables. Case 1 refers to $\Phi_{1,h} = \Phi_{2,h} = 1$, benchmark Taylor rule. Case 2 to $\Phi_{1,h}$=10, $\Phi_{2,h} = 0$, where $\Phi_{1,h}$ is associated to Home produced tradables. Case 3 to $\Phi_{1,h}$=$\Phi_{2,h}$ =1, where $\Phi_{1,h}$ is associated to the aggregate inflation index of tradables; case 4 to $\Phi_{1,h}$=10, $\Phi_{2,h} = 0$, where $\Phi_{1,h}$ is associated to the aggregate inflation index of tradables.

Figure 3.5. Calibration: $\Phi_{1,h}$ = $\Phi_{2,h}$ = $\Phi_{1,f}$ = $\Phi_{2,f}$ =1. Flexible prices in both countries.

Figure 3.6a. Calibration: $\Phi_{1,h}$= $\Phi_{2,h}$ = $\Phi_{1,f}$ =$\Phi_{2,f}$ =1. Case 1 refers to flexible prices; nominal rigidities for Home tradables only refer to case 2; nominal rigidities for Foreign tradables only refer to case 3; nominal rigidities for both Home and Foreign tradables refer to case 4.

Figure 3.6b. Calibration: $\Phi_{1,h}$= $\Phi_{2,h}$ = $\Phi_{1,f}$ = $\Phi_{2,f}$ =1. Case 1 refers to flexible prices; nominal rigidities for Home tradables only refer to case 2; nominal rigidities for Foreign tradables only refer to case 3; nominal rigidities for both Home and Foreign tradables refer to case 4.

Figure 3.7. Nominal rigidities for tradables. Case 1 refers to $\Phi_{1,h}$=$\Phi_{2,h}$ = 1, benchmark Taylor rule. Case 2 to $\Phi_{1,h}$=10, $\Phi_{2,h} = 0$, where $\Phi_{1,h}$ is associated to Home produced goods and the Foreign central bank does not target house prices, $\Phi_{2,f} = 0$. Case 3 to $\Phi_{1,h}$=$\Phi_{2,h}$ =1, where $\Phi_{1,h}$ is associated to the aggregate index for aggregate tradable inflation; case 4 to $\Phi_{1,h}$=10, $\Phi_{2,h} = 0$, where $\Phi_{1,h}$ is associated to aggregate inflation index for tradables and the Foreign

central bank does not target house prices, $\Phi_{2,f} = 0$.

Figure 3.8a. Calibration: $\Phi_{1,h}$=$\Phi_{2,h}$ = $\Phi_{1,f}$=$\Phi_{2,f}$ =1. Price rigidities in the tradable sector in both countries.

Figure 3.8b. Calibration: $\Phi_{1,h}$=$\Phi_{2,h}$ = $\Phi_{1,f}$=$\Phi_{2,f}$ =1. Case 2 corresponds to price rigidities in the tradable sector; case 1 corresponds to the benchmark flexible prices in both countries.

Figure 3.8c. Nominal rigidities for tradables in both countries; $\Phi_{1,f}$ = $\Phi_{2,f}$ =1 always. Case 1 refers to $\Phi_{1,h}$=$\Phi_{2,h}$ = 1, benchmark Taylor rule. Case 2 to $\Phi_{1,h}$=10, $\Phi_{2,h} = 0$, where $\Phi_{1,h}$ is associated to Home produced goods. Case 3 to $\Phi_{1,h}$=$\Phi_{2,h}$ =1, where $\Phi_{1,h}$ is associated to the aggregate index of inflation for tradables; case 4 to $\Phi_{1,h}$=10, $\Phi_{2,h} = 0$, where $\Phi_{1,h}$ is associated to aggregate tradables.

Figure 3.9a. Calibration: $\Phi_{1,h}$= $\Phi_{2,h}$ = $\Phi_{1,f}$ =$\Phi_{2,f}$ =1. In case 2, prices are rigid in the tradable sector, in country F. In case 1, all prices are flexible.

Figure 3.9b. Calibration: $\Phi_{1,h}$= $\Phi_{2,h}$ = $\Phi_{1,f}$ =$\Phi_{2,f}$ =1. In case 2, prices are rigid in the tradable sector, in country H. In case 1, all prices are flexible.

Figure 3.9c. Calibration: $\Phi_{1,h}$= $\Phi_{2,h}$ = $\Phi_{1,f}$ =$\Phi_{2,f}$ =1. All prices are flexible.

Figure 3.9d. Nominal rigidities for all tradables. Case 1 refers to $\Phi_{1,h}$=$\Phi_{2,h}$ = 1, benchmark Taylor rule. Case 2 to $\Phi_{1,h}$=10, $\Phi_{2,h} = 0$, where $\Phi_{1,h}$ is associated to Home produced goods and the Foreign central bank does not target house prices, $\Phi_{2,f} = 0$. Case 3 to $\Phi_{1,h}$=$\Phi_{2,h}$ =1, where $\Phi_{1,h}$ is associated to the aggregate inflation index of tradables; case 4 to $\Phi_{1,h}$=10, $\Phi_{2,h} = 0$, where $\Phi_{1,h}$ is associated to the aggregate inflation index of tradables and the Foreign central bank does not target house prices, $\Phi_{2,f} = 0$.

Figure 3.10a. Calibration: $\Phi_{1,h}$ = $\Phi_{2,h}$ = $\Phi_{1,f}$ =$\Phi_{2,f}$ =1. All prices are flexible. Variables referring to country H show the response to a productivity shock in country H; variables referring to country F refer to the response of a productivity shock in country F.

Figure 3.10b. Calibration: $\Phi_{1,h}$=$\Phi_{2,h}$ = $\Phi_{1,f}$ =$\Phi_{2,f}$ =1. All prices are flexible.

Figure 3.10c. Calibration: $\Phi_{1,h}$=$\Phi_{2,h}$ = $\Phi_{1,f}$ =$\Phi_{2,f}$ =1. In case 2, prices are rigid in the tradable sector, in country H. In case 1, all prices are flexible.

Figure 3.10d. Nominal rigidities for tradables; $\Phi_{1,f}$=$\Phi_{2,f}$ =1. Case 1 refers to $\Phi_{1,h}$ =$\Phi_{2,h}$ = 1, benchmark Taylor rule. Case 2 to $\Phi_{1,h}$=10, $\Phi_{2,h} = 0$, where $\Phi_{1,h}$ is associated to Home produced goods and the Foreign central bank does not target house prices, $\Phi_{2,f} = 0$. Case 3 to

$\Phi_{1,h}$=$\Phi_{2,h}$ =1, where $\Phi_{1,h}$ is associated to the aggregate inflation index of tradables; case 4 to $\Phi_{1,h}$=10, $\Phi_{2,h} = 0$, where $\Phi_{1,h}$ is associated to the aggregate inflation index of tradables and the Foreign central bank does not target house prices, $\Phi_{2,f} = 0$.

Chapter 4

Skills, immigration and selective policies

4.1 Introduction

In this work[1], we analyze the decision process that leads the inhabitants of destination countries to choose immigration policies that discriminate between skilled and unskilled workers, through a voting system.

Globalization creates new opportunities and challenges for financial and human capital mobility across national borders. During the last decades, as a reflection of the increasing volume of transactions, labor markets have also experienced a significant enlargement. Policy makers and international organizations have first directed their attention towards the elimination of trade barriers and the liberalization of financial markets; then, the focus has been increasingly enlarged so as to account for the implications of liberalization on human capital mobility. For instance, the approval of NAFTA was the result of a long lasting debate on the possible repercussions on employment and factors mobility in the US. Also in Europe, free factor mobility is the outcome of a process that was started to facilitate trade. Nonetheless, the enlargement of the European Union constitutes nowadays a new challenge for European immigration policies.

Immigration policies are today a subject of great political debate in destination countries.

[1] This Chapter is forthcoming as "Skills, immigration and selective policies" (2008), in *Zagreb International Review of Economics and Business* (with S. Bosi and F. Magris).

Indeed, since the collapse of the Soviet Union, only few countries restrict their citizen the possibility to migrate (such as Cuba, Myanmar and North-Korea). A part from these few exceptions, migration restrictions are decided by destination countries. Immigration flows represent both a precious resource and a cost for destination countries. Immigrants provide part of the unskilled labor force, especially in light of the old aging of the population in many industrial countries. According to the IMF (2005b), elderly dependency ratios in the advanced countries will nearly double by 2050. In addition, highly qualified immigrants can offer skills that are both scarce and critical. Finally, cultural diversity is at the root of skill complementarity and scientific progress.

However, cultural diversity is also associated with significant social costs due to the need of adapting institutions to an heterogeneous population; immigration inflows also imply an increase in public expenditure. Nannestad (2007) provides a detailed survey on the debate concerning the benefits and costs of migration inflows into welfare states. The analysis shows that the overall impact of migration on welfare crucially depends on the peculiar features of both immigrant workers and the welfare system. For the purpose of optimizing costs and benefits, immigration flows need to be regulated.

Selection criteria are nowadays an issue of great priority in the agenda of policy makers. In Australia, selection criteria are subject to a skill-assessment system and are imbedded in a well defined legislation that is articulated in several cases. The Canadian immigration system is very similar to the Australian one, while the US one is articulated less structured. Still, it is based on skills selection criteria. While the above countries have historically been mass destination countries, mass immigration flows towards Europe are a relatively recent phenomenon; in Europe, immigration policies are currently a big concern for policy makers and the enlargement of the European Union represents an additional challenge for coordinated policy-making.

Our work focuses on the decision process that leads destination countries to introduce selective immigration policies based on skill requirements. In our framework, selection criteria depend on skills:[2] each immigration policy is defined as a couple of probabilities to enter the country, that refer respectively to unskilled and skilled potential immigrants. Since any border

[2] Other authors introduce different selection criteria. For instance, in Storesletten (2000) selection depends on age.

closure implies entry rationing, a restrictive policy simply represents, from migrants' point of view, a lower probability of entering.

In the destination country, natives choose their optimal immigration policy, by evaluating the effect of immigration flows on their income. For simplicity, we assume that potential immigrants do not own physical capital. The entry of immigrants have thus a positive impact on the income of capital holders (the rich), and a negative effect on the earnings of the poor. Indeed, given initial endowments, immigration inflows raise the interest rate and dampen wages. Moreover, the entrance of highly-intensive human capital (i.e., skilled workers) exacerbates this mechanism because of its stronger positive effect on capital income and its stronger negative effect on wages.

Immigration policies are not costless. When the entrance of immigrants is followed by an increase in government spending, migration inflows imply higher taxation and have a negative effect on natives' income.[3]

If the considered framework allows also for inter-generational dynamics, the costs of immigration can differ from generation to generation. In Razin et al. (2002), redistributive policies are more costly to the native-born as immigrants share the redistribution benefits with her/him. However, since immigrants become citizens, the result of a majority vote is affected also by their preferences towards both redistribution and immigration. Dolmas and Huffman (2004) and Ortega (2005) model the voting process over immigration quotas and redistributive taxation in a dynamic setup of overlapping generations; natives' preferences for immigration today are influenced by the prospect that tomorrow immigrants will be voting over future tax policy.

In this work, we do not analyze the manifold links between immigration inflows, taxes and public spending. We focus instead on immigrations policies and their costs of implementation that we assume to be financed by the citizens of the destination country only.

In order to have a concrete idea of implementation costs, one can think about the current debate on US southern frontiers and the recent (costly) proposals to reinforce the borders. When voting over immigration policies, rational individuals take into account the costs of implementing restrictions: the stricter the restrictions, the higher the enforcement costs (free

[3]See Nannestad (2007) for an overview of the literature.

factor mobility implies zero costs). In our model, we assume for simplicity that implementation costs are financed through a flat tax on capital income and enter the utility function additively.

In absence of implementation costs, we recover Benhabib (1996) pioneering result. The political outcome over immigration policies depends on the distribution of capital: if the median voter is sufficiently rich, the optimal policy consists in keeping frontiers open; if the median voter is relatively poor, borders are kept closed.

The introduction of enforcement costs can affect this result and prevent countries' frontiers from complete closure. In presence of policy enforcement costs, Magris and Russo (2005) generalize Benhabib (1996) and obtain an interior solution; this also rules out a complete closure of borders. However, their model does not account for heterogeneity in skills.

Heterogeneity in skills is of twofold interest: it entails different degrees of complementarity between labor and physical capital and it allows to differentiate the costs of screening workers.[4]

Our work allows to explain both the emergence of immigration quotas and the existence of distinct quotas for heterogeneous immigrants. When enforcement costs are positive for both the types of immigrants, there is no room for selecting one category only; indeed, if border controls are marginally relaxed, enforcement costs significantly fall and the total impact on income is positive. Moreover, while adding realism, introducing different quotas provides an additional policy instrument to the policy maker.

Our framework allows us thus to generalize Benhabib (1996) by introducing a two-dimensional decision space (i.e. heterogeneity in skills and quotas for skilled and unskilled workers) and to extend Magris and Russo (2005) by introducing distinct enforcement costs.

The individual optimal decision is eventually incorporated in policy-making through a political decision process. All possible policies are proposed to voters, pair by pair, in a voting process *à la* Condorcet. The winner policy (i.e., the policy that wins against any possible alternative: the Condorcet winner) coincides with the choice of the median voter: the richer the median voter in terms of physical wealth, the less restrictive immigration regulations. Under mild assumptions, we show that the rich prefer skilled workers while the poor prefer the unskilled ones. On the one hand, skilled workers have a stronger positive impact on the income of

[4] One can think that enforcement costs for the skilled differ from the ones for unskilled workers, when clandestine immigrants are unskilled.

capital owners, because of a higher degree of factor complementarity; on the other hand, skilled immigrants have a stronger dampening effect on wages.

This Chapter is organized as follows. The fundamentals are presented in the next section. Section 4.3 focuses on the immigration policies and the voting system is defined in Section 4.4. Section 4.5 concludes. Technical details are gathered in Section 4.6.

4.2 The model

We consider two open economies in a static framework: the source country and the destination country (from now on, S and D, respectively). Both countries are closed in every way but one: labor is mobile across countries. Country D is characterized by a positive aggregate level both of physical and human capital, while country S by a positive level of human capital only. A consumption good is produced under constant returns to scale. In country D, it is produced by using two factors: capital and efficient units of labor. In country S, using only labor. We assume that wages in country D are strictly higher than wages in country S. Rational agents in S maximize their income and are always willing to migrate from country S to country D.

First, D-country inhabitants choose the optimal immigration policy, which takes into account its effects on capital and labor income. Then, a fraction of S-country inhabitants successfully migrates to country D. Immigration policies are an endogenous outcome of the interactions between D-country and S-country inhabitants.

4.2.1 Destination country

Country D is populated by natives who earn their income from labor and capital. As in Benhabib (1996), they are indexed by the level of capital they are endowed with, that we denote by k.[5] Each native is endowed with a unit of labor, which is supplied inelastically in a perfectly competitive labor market. An homogeneous consumption good is produced according to a CRS aggregate production function $F(K, L)$, where K and L are aggregate capital and efficient units of labor, respectively. The production function satisfies usual neoclassical assumptions. Workers' average productivity is $f(\kappa)$ where $\kappa \equiv K/L$ denotes the capital intensity.

[5] Capital endowment is the only source of heterogeneity across natives.

Assumption 1. *$f : R_+ \to R_+$ is C^2, strictly increasing and strictly concave.*

The density of natives is given by a continuous function $n(k)$ defined over $[0, \infty)$. Thus, the aggregate capital in D is given by:

$$K = \int_0^\infty n(k)\, k dk$$

while the total population is:

$$N = \int_0^\infty n(k)\, dk$$

The median voter in the native population is endowed with an amount of capital k_m solving:

$$\int_0^{k_m} n(k)\, k dk = N/2$$

The competitive interest rate, r, and the competitive wage, w, are, respectively: $r = f'(\kappa)$ and $w = w(\kappa) = f(\kappa) - f'(\kappa)\kappa$.

Without immigration, w and r are respectively: $r = f'(\kappa_0)$ and $w = w(\kappa_0)$, where $\kappa_0 \equiv K/N$, is the pre-immigration capital-labor ratio. The total pre-immigration income, ρ_k, of individual k depends upon κ_0 and k:

$$\rho_k = w(\kappa_0) + f'(\kappa_0)\, k \tag{4.1}$$

Given the static nature of the model, agents consume their whole income. Therefore, for each native, utility coincides with her/his total income; pre-immigration utilities can be thus ranked with respect to capital endowments, according to (4.1).

4.2.2 Source country

Natives of country S do not own physical capital, but are characterized by two different levels of human capital, respectively h_1 and h_2.[6] We do not focus on human capital accumulation:

[6] A depart from this assumption would not alter the migration outcome as long as, on average, newcomers in country D are endowed with less physical capital than natives.

the endowments are exogenously given and satisfy $h_1 < h_2$. We will refer to agents with human capital h_1 as unskilled workers, whereas to those with human capital h_2 as skilled workers. The human capital in country S before migration is given by $n_1 h_1 + n_2 h_2$, where n_2 refers to the number of skilled individuals and n_1 refers to the number of the unskilled ones.

In absence of migration, country-S inhabitants dispose of a linear technology converting one unit of human capital in one unit of consumption (human capital heterogeneity does not affect the technology). We will assume in the following that, even under a complete migration, the wage in country D remains higher than that earned in country S:

Assumption 2.

$$w\left(\frac{K}{N + n_1 h_1 + n_2 h_2}\right) > 1 \tag{4.2}$$

Since the utility of S-country inhabitants coincides with their labor income, Assumption 2 implies that they always try to migrate to country D. In particular, it implies that each S-native finds profitable to migrate to D for any number of other S-natives who migrate successfully.[7] Contrary to Dustmann (2001), we assume that the utility immigrants derive from consumption in country S is equal to the utility they derive from consumption in country D. Introducing different degrees of utility would not affect our main results: indeed, we focus on a two-step process of immigration policy-making and we prevent the immigrants from coming back home in the long run.

4.3 Immigration policy

Migration is a sensitive subject in every rich country. Voters are concerned in particular in those areas where immigrants have been arriving in large numbers. In Australia, immigration inflows are regulated by a well defined legislation which is articulated in 72 cases. In order to obtain a permanent visa, potential candidates aiming at working in Australia are subject to an

[7] More precisely, all the natives of the S-country will try to migrate, if the individual income in the D-country after migration is higher than the income in the S-country: $wh_i > h_i$, $i = 1, 2$. Since the wage after migration depends positively on the new capital-labor ratio and the lowest possible ratio is $K/\left(N + n_1 h_1 + n_2 h_2\right)$ (when all the natives of the S-country migrate), we obtain (4.2) as a sufficient condition.

assessment process that evaluates their skills: the higher the score, the higher the chances to enter the country. Canada is one of the world main destination countries with about 200,000 immigrants coming every year subject to the rules of its immigration system. The Canadian immigration system is similar to the Australian one and is articulated in two categories: skilled work and business immigration. The skills assessment depends on factors as: the level of education of the candidate; her/his French or English ability; her/his work experience; age; the arranged employment; adaptability. US immigration policies are based on a complicated system of visas, which is articulated in more than 60 temporary visas and some permanent ones. A part from Family Relations Visas, permanent visas are issued only as last step of a long process that starts with a job offer. "Aliens with extraordinary ability(ies)" can apply for the permanent residence permit (the Green Card). Other categories of workers need to obtain first a "labor Certification" and pass through a sophisticated bureaucratic process. Where the labor certification has demonstrated any particular type of skills shortages in US, it is possible to be granted a Green Card. However, where the workers are not officially deemed to be skilled, the process may take several years.

While the above countries have historically been mass destination countries, mass immigration flows towards Europe are a relatively recent phenomenon. In Europe, immigration policies are currently a big concern for policy makers and the enlargement of the European Union represents an additional challenge to coordinate policy-making.

While the Schengen agreement allows foreigners in Europe to freely circulate amongst the members, permanent residency is generally regulated by national policies. The debate on selective immigration policies is currently an issue of great concern and several countries are trying to introduce selection rules based on skills and labor shortages considerations. UK has recently introduced a number of new immigration visas and work category visas based on skill-selection criteria. In France, selective immigration policies are currently under debate. The focus of the discussion is on the introduction of policy regulations that would allow France to choose its immigrants according to foreigners' skills and the needs of its economy: high skills and experience in sectors with scarce labor force will be the selection criteria. In Germany, there are currently no structured selective policies; however, the worrisome scarcity of skilled labor in IT sectors has prompted the necessity to facilitate the arrival of skilled immigrants.

The above evidence suggests that immigration policies in several countries are based on skills selection criteria. Potential immigrants are generally allowed to enter destination countries according to quotas (that depend on the skills of potential newcomers). In practice, the effectiveness of immigration restrictions is weakened by the existence of illegal immigration. For simplicity, we assume that both legal and illegal immigrants earn the same wage. Thus, both legal and illegal immigrants affect the capital-labor ratio in the same way. Having said that, illegal immigration can have significant and peculiar effects on wages: the phenomenon carries indeed manifold implications. However, in our work we focus the attention on how selective policies are determined. We proxy quotas as a probability to enter successfully the destination country; this accounts for the possibility that individuals may entry illegally.

In general, the optimality of an immigration policy critically depends on the perspective one adopts. Clearly, if we assumed the point of view of a source country, the migration of skilled workforce could be a matter of concern when associated with a loss of human capital;[8] on the other hand, the benefits deriving from remittances should also be considered. According to World Bank data, in some very poor countries remittances account nowadays for about the 50% of GDP (The Economist, 2008). Indeed, costs and benefits need to be optimally balanced.

Since we assume the perspective of destination countries, we aim at optimizing their welfare only. In our model, welfare is proxied by the revenue of its citizens[9]. Each citizen makes her/his optimal choice (i.e., chooses the optimal policy) by maximizing her/his income. Eventually, the immigration policy is the result of aggregating citizens' preferences through the voting system.

We define the immigration policy chosen in the D-country as a vector $\pi \equiv (\pi_{1,}\pi_2)$ belonging to $[0,1] \times [0,1]$, which for every $i = 1,2$ fixes the probability π_i of a successful migration for a candidate migrant with human capital endowment h_i. For simplicity, we can describe the model as a two-step process with the following timing: (1) natives chose an immigration policy π, (2) nature randomly chooses a fraction π_i, $i = 1,2$ of successful migrants of type h_i.

[8]For some discussion about the brain-drain danger, see among others Beine et al. (2001), Bhagwati and Hamada (1974), Carrington and Detriagiache (1998), Reichlin and Rustichini, (1999), Schiff (2005), Wong and Yip (1999). See also Mountford (1997), Stark et al. (1997) and Stark and Wang (2001) for some discussion on possible "brain gains" related to migration.

[9]For a focus on the impact of migration inflows see Borjas (1994). For a recent survey on the effects of immigration into welfare states see Nannestad (2007).

Aggregate labor supply and the capital-labor ratio after migration in the D-country become:

$$
\begin{aligned}
L &= N + \pi_1 n_1 h_1 + \pi_2 n_2 h_2 \\
\kappa &\equiv K/L = K/\left(N + \pi_1 n_1 h_1 + \pi_2 n_2 h_2\right)
\end{aligned} \tag{4.3}
$$

When the fundamentals are given (exogenous distributions of physical and human capital in both countries) the capital per unit of labor only depends on the immigration policy π: $\kappa = \kappa(\pi)$. Finally, notice that $\kappa_0 = \kappa(0)$. The increase in immigration quotas lowers the capital-labor ratio $\kappa(\pi)$, raises capital income and dampens labor wages.

4.3.1 Immigration policy without enforcement costs

Assume that no enforcement costs are associated to the implementation of any policy π. It follows that, for a given immigration policy π, the income of an individual endowed with an amount of capital k is given by:

$$
\rho_k(\pi) \equiv w(\kappa(\pi)) + kf'(\kappa(\pi)) \tag{4.4}
$$

An individual endowed with k maximizes (4.4) with respect to π. The optimal choice of each agent is a function of her/his endowments of capital: given factor complementarity, the richer (in capital endowments) the agent, the greater the incentive to let immigrants coming in.

Arrow's (1963) Impossibility Theorem pointed out the limits of preferences aggregation in a world populated by heterogeneous agents. In order to avoid Arrow's paradox and find an optimal policy, we need to define a simple voting mechanism in country D. In a voting procedure *à la* Condorcet, immigration policies are compared two by two. This mechanism selects the policy that wins against any other alternative, pair by pair, and assures that the larger share of the population is satisfied. The political outcome coincides with the choice of the median voter. Such an optimal policy is defined as the Condorcet winner.

Let now

$$
\tilde{k} \equiv \frac{w(\kappa(0,0)) - w(\kappa(1,1))}{f'(\kappa(1,1)) - f'(\kappa(0,0))} \tag{4.5}
$$

The following Proposition identifies the Condorcet winner in absence of implementation costs and generalizes Benhabib's (1996) result by considering a two-dimensional decision space. As in Benhabib (1996), the optimal policy depends on k_m, the median voter's capital endowment.

Proposition 3 *Without enforcement costs, the Condorcet winner is* $\pi^{**} = (0,0)$ *if* $k_m < \tilde{k}$*, and* $\pi^{**} = (1,1)$ *if* $k_m > \tilde{k}$*, where* k_m *denotes the median voter's capital endowment.*

Proof. See the Appendix.

Proposition 3 proves that, if the median voter is rich in capital (i.e., richer than a benchmark level, $\tilde{k}$), the optimal policy in the destination country consists in welcoming all potential immigrants; to the contrary, if the median voter is poor, frontiers are kept closed.

This result can help to understand why capital accumulation and technical progress have eventually prompted frontier openness and challenged protectionist mainstreams. Economic growth, sustained by endogenous technical progress, raises capital intensity and promotes the opening of borders.

4.3.2 Immigration policy with enforcement costs

According to Ortega (2004), individuals choose their optimal immigration policy taking into account its impact on future generations. Their optimization problem considers the political costs related to the entrance of immigrants endowed with complementary skills.

We will instead assume that mitigating the flows of immigrants is itself costly: the stricter the restrictions, the higher the costs. Stricter restrictions require more controls, and thus, more public expenditure. One can have an idea of these costs thinking of tighter controls at the frontiers or more infrastructures to delimit borders.

We also suppose that costs are additive in the two components of the migration policy. It is in fact reasonable to think that the suitability of unskilled workers cannot be evaluated according to the same criteria used for skilled immigrants; applications for different visas are indeed evaluated according to different protocols and, generally, by different directorates. Moreover, clandestine workers are often unskilled. Monitoring illegal flows and carrying illegal immigrants beyond the borders, possibly in the original country, is very expensive. We assume that enforcement costs are determined as follows.

$$C(\pi) \equiv C_1(\pi_1) + C_2(\pi_2)$$

For each $i = 1, 2$, the function $C_i(\pi_i)$, satisfies the following properties:

Assumption 3. *$C_i(0) > 0$, $C_i(1) = 0$, $C_i'(1) = 0$ and $C_i'(\pi_i) < 0$, $C_i''(\pi_i) > 0$ for every $\pi_i \in (0, 1)$, $i = 1, 2$.*

Assumption 4. $C_i'(0) = -\infty$, $i = 1, 2$

$C_i(0) > 0$ means that the costs of a complete closure are positive, while $C_i(1) = 0$ says that no restrictions for a given type of immigrant yields zero costs. Condition $C_i'(\pi_i) < 0$ ensures that the enforcement cost is decreasing in each of its arguments. $C_i''(\pi_i) > 0$ states the convexity of the cost: the progressive closure of frontiers entails increasing marginal costs.

Conditions $C_i'(0) = -\infty$ is sufficient to rule out complete closure. Eventually notice that $C_i'(0) = -\infty$ is a less restrictive assumption than $C_i(0) = +\infty$: complete frontier closure is technically feasible, but not economically rational.

Individual preferences

The immigration policy is financed by a flat tax on capital income:[10] $C(\pi) = \tau f'(\kappa) K$, where κ is given by (4.3) and τ is the constant tax rate. The amount of tax paid by an individual owing an amount of capital equal to k is therefore: $c_k(\pi) = \tau f'(\kappa) k = C(\pi) k/K$.

On aggregate, natives' capital income is sufficient to entirely finance the equilibrium policy. Under the above assumptions, the income of a native endowed with k in the D-country is:

$$\sigma_k(\pi) \equiv \rho_k(\pi) - c_k(\pi) = w(\kappa(\pi)) + kf'(\kappa(\pi)) - C(\pi) k/K \qquad (4.6)$$

[10] Taxing also wages would not affect the main results as long as D-country natives supply the same amount of labor.

Let now

$$\underline{k} \equiv \max_i \max_{\pi_j} \frac{n_i h_i f''(\kappa_i)\kappa_i^3}{C_i'(0)+n_i h_i f''(\kappa_i)\kappa_i^2}, \quad \text{with } \kappa_i(\pi_j) \equiv \frac{K}{N+\pi_j n_j h_j}$$
$$\bar{k} \equiv \min_i \min_{\pi_j} \frac{n_i h_i f''(\kappa_i)\kappa_i^3}{C_i'(1)+n_i h_i f''(\kappa_i)\kappa_i^2}, \quad \text{with } \kappa_i(\pi_j) \equiv \frac{K}{N+n_i h_i+\pi_j n_j h_j}$$

and the following assumption holding:

Assumption 5. $\underline{k} < \bar{k}$.

Assumption 5 is not merely a technical assumption: it defines an interval for individual capital. Clearly, the above inequality is satisfied only in presence of enforcement costs.

We first consider agents with small capital endowments whose optimal policy in the absence of costs was $(0, 0)$; one can note that in presence of marginal implementation costs, their optimal immigration policy may change. They will in fact choose to depart from the initial optimal policy so as to avoid part of the costs. However, if the marginal cost in zero is very high, a very small departure from the policy $(0, 0)$ will dramatically increase their revenues: the higher the marginal cost, the more significant the increase in income. Moreover, the lower bound of the interval of capital endowments including interior solutions is lower as soon as the marginal cost in zero is higher.

As we will prove in the next proposition, a native with capital endowments between $\underline{k}$ and $\bar{k}$ will choose an interior solution for program (4.6) ($0 < \pi_i^* < 1$, $i = 1, 2$). On the one hand, a native who is endowed with capital $k > \underline{k}$ does not find profitable to vote for complete closure (notice that Assumption 4 entails $\underline{k} = 0$, that is, always $k > \underline{k}$). On the other hand, a capital endowment below $\bar{k}$ prevents the native from choosing complete openness.

Proposition 4 *Under the Assumption 1, 3 and 5, the optimal immigration policy* $\arg\max_\pi \sigma_k(\pi)$ *for the individual* $k \in (\underline{k}, \bar{k})$ *is an interior solution* $\pi_k^* \in (0, 1) \times (0, 1)$.

Proof. See the Appendix.

We observe that Assumption 4 is not required in the proof of Proposition 4: for $k < \underline{k}$, the case of a poor individual who chooses complete closure is not ruled out. When $\underline{k} < k < \bar{k}$, a middle-class native vote**s** a policy-mix consisting of allowing (at least) some immigrants (but not all workers willing to enter) coming in.

Numerical example Let us provide a numerical illustration of the results in Proposition 4. In the following, we relax Assumption 4, so as to allow for the poor to choose complete frontier closure. For simplicity, we assume that output is determined according to a Cobb-Douglas technology with reduced form:

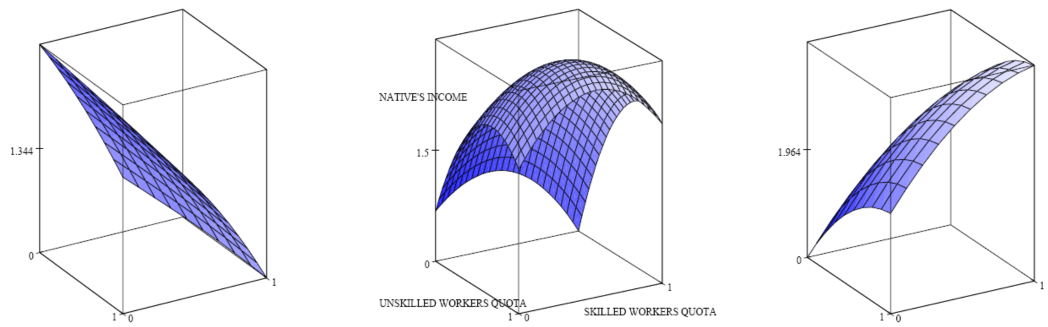
$$f(\kappa) = \kappa^{\alpha} \tag{4.7}$$

It follows that $r = \alpha\kappa^{\alpha-1}$, $w = (1-\alpha)\kappa^{\alpha}$. Assuming also that $C_i(\pi_i) \equiv \pi_i^2/2 - \pi_i + 1/2$, $i = 1, 2$, we obtain

$$\sigma_k(\pi) = \alpha k \kappa(\pi)^{\alpha-1} + (1-\alpha)\kappa(\pi)^{\alpha} - \left[\left(\pi_1^2 + \pi_2^2\right)/2 - \pi_1 - \pi_2 + 1\right] k/K$$

where $\kappa(\pi)$ is given by (4.3).

Setting $\alpha = 1/3$, $K = 32$, $N = 4$, $n_1 = n_2 = 1$, $h_1 = 1/2$, $h_2 = 1$, we plot the income function:



The poor: $k = 1$ Middle class: $k = 3$ The rich: $k = 9$

Figure 4.1. Capital and income function

We observe that the poor and the rich choose complete closure and complete openness, respectively. Individual with intermediate endowments of capital (denoted as middle class) prefers an interior solution for both quotas. For certain levels of capital endowments, the individual income optimization process implies interior solutions for both immigration quotas π_1 and π_2. This result fits realistically the empirical evidence and contributes to explain why selective policies generally imply positive quotas both for skilled and unskilled workers. The above results suggest also that the individual's optimal degree of frontier openness depends on her/his capital endowments.

Given the heterogeneity of capital endowments, natives are subject to diverse incentives and have different preferences toward the immigration policy. Before aggregating them, we thus need to prove the existence of a ranking of preferences which depends upon capital.

We now analyze the sensitivity of individual preferences with respect to wealth. In the following, the elasticity

$$\varepsilon_i \equiv (k/\pi_i^*)\, d\pi_i^*/dk \tag{4.8}$$

captures how the individual's preferences for openness varies with her/his wealth. The second-order elasticity

$$\gamma_i(\pi_i) \equiv \pi_i C_i''(\pi_i)/C_i'(\pi_i) < 0 \tag{4.9}$$

resumes enforcement costs' convexity. Finally,

$$\nu_i \equiv \pi_i n_i h_i / (N + \pi_1 n_1 h_1 + \pi_2 n_2 h_2) \in [0, 1) \tag{4.10}$$

denotes the share of skilled-workers in total human capital.

Consider now the inhabitants of country D. As previously set, the only source of heterogeneity is given by their endowment in physical capital. We thus proceed by characterizing D-country inhabitants' immigration preferences according to their endowment in physical capital. For simplicity, we introduce the following notation: $\pi < \pi'$ iff $\pi_i < \pi_i'$ for $i = 1, 2$.

It is possible to prove that the best immigration policy is non-decreasing in the capital endowment of each native. We know that π_k^* (where π_k^* denotes an optimal policy for the individual k) exists and, under the conditions of Proposition 4, it is interior. We need to prove that individuals endowed with more capital prefer to increase frontier openness so as to attract a more complementary factor. The first step is proving the following lemma.

Lemma 5 *Under Assumptions 1 and 3, signum* $\partial\pi_{1k}^*/\partial k =$ *signum* $\partial\pi_{2k}^*/\partial k$*. Moreover, if*

$\bar{\xi} < \xi < 0$ or $0 < \xi < \bar{\xi}$, where

$$\xi \equiv \kappa / (k - \kappa) \tag{4.11}$$

$$\bar{\xi} \equiv 2 + \varphi + (\nu_1/\gamma_1 + \nu_2/\gamma_2)^{-1} \tag{4.12}$$

and $\varphi(\kappa) \equiv \kappa f'''(\kappa) / f''(\kappa)$, then

$$\partial \pi_{ik}^* / \partial k > 0 \tag{4.13}$$

$i = 1, 2$.[11]

A stationary point such that $\partial \pi_{ik} / \partial k = 0$ for $i = 1, 2$ is a local maximum π_{ik}^* if $\xi > \bar{\xi}$ and a saddle point if $\xi < \bar{\xi}$.

Proof. See the Appendix.

Lemma 5 allows us to individuate a set of optimal choices, which is an upward-sloped curve in the π-plane; π_{2k}^* is in fact a strictly increasing function of π_{1k}^*. In addition, this curve points at $\pi = (1, 1)$ and, if the conditions for ξ in Lemma 5 are satisfied, the individually preferred policy moves to $(1, 1)$ as k increases. Lemma 6 clarifies the link between the individual optimal policy and capital endowments.

Lemma 6 *Let $\pi < \pi'$ and $k_1 < k_2$. Then*

$$\sigma_{k_2}(\pi) > \sigma_{k_2}(\pi') \text{ implies } \sigma_{k_1}(\pi) > \sigma_{k_1}(\pi') \tag{4.14}$$

$$\sigma_{k_1}(\pi') > \sigma_{k_1}(\pi) \text{ implies } \sigma_{k_2}(\pi') > \sigma_{k_2}(\pi) \tag{4.15}$$

Proof. See the Appendix.

Lemma 6 highlights a non-decreasing relation between the physical wealth and the degree of frontier openness: if the poor prefer to increase frontier openness, the rich *a fortiori* do; if the rich prefer to strengthen frontier tightness, the poor *a fortiori* do. The reason is that an increase in labor dampens the capital-labor ratio and raises capital income.

---

[11] *In the Cobb-Douglas case (3.9), the elasticity φ and the critical point $\bar{\xi}$ simplify to $\varphi = \alpha - 2 \in (-2, -1)$ and $\bar{\xi} \equiv \alpha + (\nu_1/\gamma_1 + \nu_2/\gamma_2)^{-1}$, respectively.*

The following assumption allows to analyze the relation between wealth and openness and individuate a positive link.

Assumption 6. *The costs are isoelasic (i.e., γ_i's are constant) and sufficiently convex, (i.e., $\gamma_i < -\nu_i (2 + \varphi - \xi)$* for $i = 1, 2$).

The following Proposition allows us to conclude that individuals endowed with more capital prefer frontier openness.

Proposition 7 *Under Assumptions 1, 3 and 6, the individual solution increases with the capital endowment: $k_2 > k_1$ implies $\pi^*_{ik_2} > \pi^*_{ik_1}$ with $i = 1, 2$.*

Proof. See the Appendix.

Aggregated preferences (the Condorcet winner)

In the above sections we have individuated, for each agent, the relation between her/his capital endowments and her/his preferences over immigrations. In order to define the immigration policy that will be implemented, preferences need to be aggregated. Immigration policies are an issue of great debate in most of industrial countries. The reason why the public opinion is intensively involved in the discussion is that their effects are immediate and visible.

We now show that, as in the case with no implementation costs, the median voter sets the optimal policy.

Proposition 8 *Under the Assumptions 1, 3, 6, the median voter's choice is the Condorcet winner.*

Proof. Consider now the median voter's optimal immigration policy, $\pi^*_{k_m}$, and compare it with any $\pi_a > \pi^*_{k_m}$. Since the median voter is the richest of all the voters on her/his left, implication (4.14) holds and $\pi^*_{k_m}$ is voted by these voters and the median voter her/himself, that is, by a majority.

Consider now the comparison between $\pi^*_{k_m}$ and any $\pi_b < \pi^*_{k_m}$. The median voter is poorer than any voters on her/his right. In this case, the implication (4.15) is verified and $\pi^*_{k_m}$ is voted by a majority (median voter and voters on the right).

Therefore, $\pi^*_{k_m}$ is the Condorcet winner. ■

The following proposition shows that, under a plausible assumption[12], if the poor prefers skilled immigrants, the rich has an even stronger bias for skilled workers; instead, if the rich prefers unskilled immigrants, the poor has a stronger preference for unskilled workers. The following Proposition highlights that rich natives have a sort of relative preference for skilled immigrants, while, on the contrary, the poor have a preference for unskilled immigrants.

Proposition 9 *Assume (4.13) holds and $\gamma_1 < \gamma_2$. If a given individual h chooses at optimum $\pi^*_{\hat{k}}$, with $\pi^*_{1\hat{k}} < \pi^*_{2\hat{k}}$, then all individuals with $k > h$ will choose $\pi^*_{1k} < \pi^*_{2k}$. Symmetrically, if the individual h chooses at optimum π^*_h, with $\pi^*_{1h} > \pi^*_{2h}$, then all individuals with $k < h$ will choose $\pi^*_{1k} > \pi^*_{2k}$.*

Proof. $\gamma_1 < \gamma_2$ implies $1/\gamma_1 > 1/\gamma_2$ and, under the conditions ensuring (4.13), we get $\varepsilon_1 < \varepsilon_2$ or, equivalently, we obtain $\varepsilon_2 - \varepsilon_1 = d\ln(\pi_2/\pi_1)/d\ln k > 0$. If $\pi^*_1 < \pi^*_2$, then a higher capital endowment increases the gap $\pi^*_2 - \pi^*_1$. ■

4.4 Concluding remarks

This work focuses on the process that leads destination countries to implement selective immigration policies based on skill selection criteria. If the enforcement of the immigration policy is not associated with costs, we recover Benhabib (1996) pioneering result and we generalize it so as to account for immigrants heterogeneity in skills. As in Benhabib (1996), we find that individuals' preferences are polarized between complete frontier openness and complete closure according to their capital endowments. Preferences are then aggregated through a voting system *à la* Condorcet and all immigration policies are confronted, couple by couple. The resulting policy, the Condorcet winner, depends on the capital endowment of the median voter: if s/he is rich, frontiers are let completely open. If s/he is poor, they are kept closed.

Policy implementation costs can affect this result and allow for interior solutions. Implementation costs are a positive function of the associated border controls and a burden on natives' income: the stricter the policy, the higher the costs. We show that in presence of enforcement

[12] $\gamma_1 < \gamma_2$ means that a more restrictive regulation entails higher marginal costs for unskilled than for skilled workers.

costs there is a range of agents who prefer to let some (but not all) potential immigrants come in. By accounting for different costs and skill heterogeneity, our framework extends Magris and Russo (2005). We show that skill heterogeneity implies different degrees of capital-labor complementarity; native capital-owners find profitable to attract highly intensive labor. To the contrary, poor natives prefer unskilled workers.

We prove that for an range of capital endowments, natives vote distinct and interior quotas both for skilled and unskilled workers. This interval of capital endowments (and thus, of individuals) increases with the marginal costs of policy implementation. Preferences are eventually aggregated through a voting system *à la* Condorcet. The policy outcome depends on the median voter: the richer, the larger the shares of welcomed workers.

4.5 Appendix

Proof of Proposition 3. In order to characterize the solution, it is useful to compute the partial derivatives of $\rho_k(\pi)$. Since $w'(\kappa) = -\kappa f''(\kappa)$, we obtain $\rho_{ki} = (k-\kappa) f''(\kappa)\,\partial\kappa/\partial\pi_i$. Noticing that $f''(\kappa) < 0$ and that $\partial\kappa/\partial\pi_i = -n_i h_i \kappa/L < 0$, we immediately verify that $\partial\rho_k/\partial\pi_i > 0$ iff $k > \kappa$ and both the derivatives vanish at $k = \kappa$.

The implicit inequality $k \geq \kappa(\pi)$ allow us to determine from (4.3) a one-dimensional line in the (π_1, π_2)-plane:

$$\pi_2 \geq a_k - b\pi_1 \tag{4.16}$$

with $a_k \equiv (1/k - 1/\kappa_0) K/(n_2 h_2)$ and $b \equiv n_1 h_1/(n_2 h_2)$.

Then $\partial\rho_k/\partial\pi_i > 0$ iff $\pi_2 > a_k - b\pi_1$, $i = 1, 2$. Since $b > 0$, the sign of the derivative crucially depends upon the value of a_k.

Assume first $a_k \in [0, 1+b]$. This implies that ρ_k is minimal along the negative-sloped line (4.16) passing between $(0,0)$ and $(1,1)$. Below this line both the derivatives $\partial\rho_k/\partial\pi_i$ are negative; since the function ρ_k inherits the property C^1 from the assumption $f \in C^2$, then $\pi = (0,0)$ is the $\arg\max \rho_k$. Similarly, above the line (4.16) both the derivatives are positive; also, since the function ρ_k is C^1, then $\pi = (1,1)$ is the $\arg\max \rho_k$. The global maximum in the unit square $[0,1] \times [0,1]$ is the maximum of these two maxima, but the optimal choice remains one of the corner solutions, $(0,0)$ or $(1,1)$: $\pi_k^* = \arg\max\{\rho_k(0,0), \rho_k(1,1)\}$.

If $a_k < 0$, then $\pi_k^* = (1,1)$. Finally, if $a_k > 1+b$, then $\pi_k^* = (0,0)$.

We now need to prove that the population is polarized between those preferring $\pi = (0,0)$, and those preferring $\pi = (1,1)$. Consider type $\tilde{k}$, who is indifferent between these polar choices, i.e. the type that solves $\rho_k(0,0) = \rho_k(1,1)$ or, more explicitly, expression (4.5).

We observe that $\tilde{k} > 0$ because $\kappa(0,0) > \kappa(1,1)$, that is $w(\kappa(0,0)) > w(\kappa(1,1))$ and $f'(\kappa(0,0)) < f'(\kappa(1,1))$. Such a type exists and is unique, provided that there is at least a native with a sufficiently low capital endowment ($k < \tilde{k}$) and another with a sufficiently large capital endowment ($k > \tilde{k}$).

First notice that $k < \tilde{k}$ iff $\rho_k(0,0) > \rho_k(1,1)$. In other terms, low capital types $k < \tilde{k}$ will prefer $\pi = (0,0)$, while high capital types $k > \tilde{k}$ will prefer $\pi = (1,1)$. Therefore if $k_m < \tilde{k}$ ($k_m > \tilde{k}$), the policy $\pi = (0,0)$ ($\pi = (1,1)$) is a Condorcet winner. ■

Proof of Proposition 4. In order to find the optimal immigration policy for individual k, we compute now the partial derivatives of (4.6):

$$\frac{\partial \sigma_k}{\partial \pi_i} = -\frac{k}{K}\left[n_i h_i \kappa^2 f''(\kappa)\left(1-\frac{\kappa}{k}\right) + C_i'(\pi_i)\right] \tag{4.17}$$

for $i = 1,2$. It is then possible to find the conditions under which σ_k has an interior optimum for π_i, given the value of π_j, $i \neq j$. For this purpose, notice that $\lim_{\pi_i \to 0} \partial\sigma_k/\partial\pi_i > 0$, when

$$k > \underline{\mathrm{k}}_i(\pi_j) \equiv \kappa_i \frac{n_i h_i \kappa_i^2 f''(\kappa_i)}{n_i h_i \kappa_i^2 f''(\kappa_i) + C_i'(0)} \ (\geq 0) \tag{4.18}$$

where $\kappa_i(\pi_j) \equiv K/(N + \pi_j n_j h_j)$.

Consider $\underline{\mathrm{k}}_i$ as a function $\underline{\mathrm{k}}_i = \underline{\mathrm{g}}_i(\kappa_i)$ and observe that, since

$$\kappa_i([0,1]) = \left[\frac{K}{N + n_j h_j}, \frac{K}{N}\right]$$

is a compact set and $\underline{\mathrm{g}}_i$ is continuous in $\kappa_i([0,1])$, then $\underline{\mathrm{k}}_i$ attains a maximum value $\max_{\pi_j} \underline{\mathrm{k}}_i(\pi_j)$. Thus, if $k > \max_{\pi_j} \underline{\mathrm{k}}_i(\pi_j)$, then $\lim_{\pi_i \to 0} \partial\sigma_k/\partial\pi_i > 0$, for each $\pi_j \in [0,1]$.

Let now

$$\underline{\mathrm{k}} \equiv \max\left\{\max_{\pi_2} \underline{\mathrm{k}}_1(\pi_2), \max_{\pi_1} \underline{\mathrm{k}}_2(\pi_1)\right\} (\geq 0) \tag{4.19}$$

It follows that $\lim_{\pi_i \to 0} \partial\sigma_k/\partial\pi_i > 0$, for $i = 1, 2$ and for all $k > \underline{k}$.

Similarly, we need to find the condition under which $\lim_{\pi_i \to 1} \partial\sigma_k/\partial\pi_i < 0$ for $i = 1, 2$. According to expression (4.17), this happens when

$$k < \bar{k}_i(\pi_j) \equiv \kappa_i \frac{n_i h_i \kappa_i^2 f''(\kappa_i)}{n_i h_i \kappa_i^2 f''(\kappa_i) + C_i'(1)} \ (\geq 0) \tag{4.20}$$

where $\kappa_i(\pi_j) \equiv K/(N + n_i h_i + \pi_j n_j h_j)$.

As above, consider $\bar{k}_i$ as a function $\bar{k}_i = \bar{g}_i(\kappa_i)$ and observe that, since

$$\kappa_i([0,1]) = \left[\frac{K}{N + n_1 h_1 + n_2 h_2}, \frac{K}{N + n_i h_i}\right]$$

is a compact set and $\bar{g}_i$ is continuous in $\kappa_i([0,1])$, then $\bar{k}_i$ attains a minimum value $\min_{\pi_j} \bar{k}_i(\pi_j)$. Thus, if $k < \min_{\pi_j} \bar{k}_i(\pi_j)$, we have $\lim_{\pi_i \to 1} \partial\sigma_k/\partial\pi_i < 0$ for every $\pi_j \in [0,1]$.

Setting

$$\bar{k} \equiv \min\left\{\min_{\pi_2} \bar{k}_1(\pi_2), \min_{\pi_1} \bar{k}_2(\pi_1)\right\} (\geq 0) \tag{4.21}$$

we get that $\lim_{\pi_i \to 1} \partial\sigma_k/\partial\pi_i < 0$ for $i = 1, 2$ for all $k < \bar{k}$.

Then, if $\underline{k} < \bar{k}$ and $k \in (\underline{k}, \bar{k})$, for each point located on the frontier of the unit square $[0,1] \times [0,1]$, there exists at least one strictly preferred point in the interior $(0,1) \times (0,1)$ of the unit square. Moreover, the objective function σ_k, (4.6), is bounded from above and, under Assumptions 1 and 3, it is C^1 on $[0,1] \times [0,1]$. Therefore, the function attains a maximum in the interior $(0,1) \times (0,1)$. ■

Proof of Lemma 5. The individual k's optimal solution π_k^* is solution of the two-dimensional system $\partial\sigma_k/\partial\pi_i = 0$, $i = 1, 2$. Using (4.17), we obtain a set of two equations

$$n_i h_i \kappa^2 f''(\kappa)(1 - \kappa/k) + C_i'(\pi_i) = 0 \tag{4.22}$$

$i = 1, 2$. We are interested in the impact of k on π_k^*. In order to compute the derivatives $\partial\pi_{ik}^*/\partial k$, we apply the implicit function theorem to system (4.22). More explicitly, we compute the total differential of system (4.22) with respect to k, π_1, π_2. Using (4.3) and still (4.22), we get

$$\left(\left[2+\varphi\left(\kappa\right)-\xi\right]\nu_i+\gamma_i\left(\pi_i\right)\right)\frac{d\pi_i}{\pi_i}+\left[2+\varphi\left(\kappa\right)-\xi\right]v_j\frac{d\pi_j}{\pi_j}=\xi\frac{dk}{k} \tag{4.23}$$

$i=1,2,\ i\neq j$, where $\gamma_i\left(\pi_i\right)$, ν_i and ξ are given by (4.9), (4.10) and (4.11), respectively, while $\varphi\left(\kappa\right)$ is defined in Lemma 5.

Introducing elasticities (4.8), system (4.23) simplifies to $\left[\left(2+\varphi-\xi\right)\nu_i+\gamma_i\right]\varepsilon_i+\left(2+\varphi-\xi\right)\nu_j\varepsilon_j=\xi$, where $i=1,2,\ i\neq j$.

Solving this system, we obtain the impact of k on π_k^* in terms of elasticities:

$$\left[\begin{array}{c}\varepsilon_1\\ \varepsilon_2\end{array}\right]=\frac{\xi}{1-\frac{2+\varphi-\xi}{2+\varphi-\bar{\xi}}}\left[\begin{array}{c}1/\gamma_1\\ 1/\gamma_2\end{array}\right] \tag{4.24}$$

where $\bar{\xi}$ is given by (4.12).

Notice first that signum $\varepsilon_1=$ signum ε_2. This implies that the impact of capital on π_{1k}^* is positive iff the impact on π_{2k}^* is positive. According to (4.24), we determine conditions under which capital has a positive impact on π^*, i.e. (1) if $k<\kappa$, that is $\xi<0$, we require $\bar{\xi}<\xi<0$; (2) if $k>\kappa$, that is $\xi>0$, we require $0<\xi<\bar{\xi}$ Equivalently, we have two possible cases for a positive impact of k on π_k^*: (1) if $\bar{\xi}<0$, we require $\bar{\xi}<\xi<0$; (2) if $\bar{\xi}>0$, we require $0<\xi<\bar{\xi}$.

Let us now characterize the concavity of $\sigma_k\left(\pi\right)$ in a neighborhood of π_k^*. We compute the Hessian matrix of 4.6 evaluated at the stationary point. From (4.17), the first order conditions (4.22) and $\partial\kappa/\partial\pi_i=-n_ih_i\kappa/L$, we obtain:

$$\left[\begin{array}{cc}\frac{\partial^2\sigma_k}{\partial\pi_1^2} & \frac{\partial^2\sigma_k}{\partial\pi_1\partial\pi_2}\\ \frac{\partial^2\sigma_k}{\partial\pi_2\partial\pi_1} & \frac{\partial^2\sigma_k}{\partial\pi_2^2}\end{array}\right]_{\pi_k=\pi_k^*}$$

$$=\left[\begin{array}{cc}-\left[\gamma_1+\nu_1\left(2+\varphi-\xi\right)\right]\frac{k}{K}\frac{C_1'(\pi_1)}{\pi_1} & -\left(2+\varphi-\xi\right)\frac{k}{\kappa}\frac{n_2h_2C_1'(\pi_1)}{L^2}\\ -\left(2+\varphi-\xi\right)\frac{k}{\kappa}\frac{n_1h_1C_2'(\pi_2)}{L^2} & -\left[\gamma_2+\nu_2\left(2+\varphi-\xi\right)\right]\frac{k}{K}\frac{C_2'(\pi_2)}{\pi_2}\end{array}\right]$$

where γ_i and ν_i are given by (4.9) and (4.10).

Under Assumptions 1 and 3, $\partial^2\sigma_k/\partial\pi_1^2$ is positive iff $\xi<2+\varphi+\gamma_1/\nu_1$, while the determinant of the Hessian matrix is positive iff $\xi>\bar{\xi}$, where $\bar{\xi}$ is given by (4.12). We observe that $2+\varphi+\gamma_1/\nu_1<\bar{\xi}$. Then $\xi>\bar{\xi}$ implies the negative definition, while $\xi<\bar{\xi}$ entails a negative determinant. In other words, the stationary point is a local maximum if $\xi>\bar{\xi}$ and a saddle point if $\xi < \bar{\xi}$. ■

Proof of Lemma 6. Suppose the hypothesis in (4.14) is true. More explicitly:

$$\begin{aligned} & w(\kappa(\pi)) + f'(\kappa(\pi))k_2 - [C_1(\pi_1) + C_2(\pi_2)]k_2/K \\ > \ & w(\kappa(\pi')) + f'(\kappa(\pi'))k_2 - [C_1(\pi'_1) + C_2(\pi'_2)]k_2/K \end{aligned}$$

or, equivalently,

$$\begin{aligned} & w(\kappa(\pi)) - w(\kappa(\pi')) + [f'(\kappa(\pi)) - f'(\kappa(\pi'))]k_2 \\ & + [C_1(\pi'_1) - C_1(\pi_1) + C_2(\pi'_2) - C_2(\pi_2)]k_2/K \\ > \ & 0 \end{aligned}$$

We notice that:

$$\begin{aligned} w(\kappa(\pi)) - w(\kappa(\pi')) &> 0 \\ f'(\kappa(\pi)) - f'(\kappa(\pi')) &< 0 \\ C_1(\pi'_1) - C_1(\pi_1) + C_2(\pi'_2) - C_2(\pi_2) &< 0 \end{aligned}$$

Then, since $k_1 < k_2$, it must be:

$$\begin{aligned} & w(\kappa(\pi)) - w(\kappa(\pi')) + [f'(\kappa(\pi)) - f'(\kappa(\pi'))]k_1 \\ & + [C_1(\pi'_1) - C_1(\pi_1) + C_2(\pi'_2) - C_2(\pi_2)]k_1/K \\ > \ & 0 \end{aligned}$$

which implies the RHS in (4.14):

$$\begin{aligned} & w(\kappa(\pi)) + f'(\kappa(\pi))k_1 - [C_1(\pi_1) + C_2(\pi_2)]k_1/K \\ > \ & w(\kappa(\pi')) + f'(\kappa(\pi'))k_1 - [C_1(\pi'_1) + C_2(\pi'_2)]k_1/K \end{aligned}$$

Suppose, now, the hypothesis in (4.15) holds. Then we have:

$$\begin{aligned} & w\left(\kappa\left(\pi'\right)\right)+f'\left(\kappa\left(\pi'\right)\right)k_1-\left[C_1\left(\pi_1'\right)+C_2\left(\pi_2'\right)\right]k_1/K \\ > \; & w\left(\kappa\left(\pi\right)\right)+f'\left(\kappa\left(\pi\right)\right)k_1-\left[C_1\left(\pi_1\right)+C_2\left(\pi_2\right)\right]k_1/K \end{aligned}$$

or, equivalently,

$$\begin{aligned} & w\left(\kappa\left(\pi'\right)\right)-w\left(\kappa\left(\pi\right)\right)+\left[f'\left(\kappa\left(\pi'\right)\right)-f'\left(\kappa\left(\pi\right)\right)\right]k_1 \\ & +\left[C_1\left(\pi_1\right)-C_1\left(\pi_1'\right)+C_2\left(\pi_2\right)-C_2\left(\pi_2'\right)\right]k_1/K \\ > \; & 0 \end{aligned}$$

Notice that:

$$\begin{aligned} w\left(\kappa\left(\pi'\right)\right)-w\left(\kappa\left(\pi\right)\right) &< 0 \\ f'\left(\kappa\left(\pi'\right)\right)-f'\left(\kappa\left(\pi\right)\right) &> 0 \\ C_1\left(\pi_1\right)-C_1\left(\pi_1'\right)+C_2\left(\pi_2\right)-C_2\left(\pi_2'\right) &> 0 \end{aligned}$$

Then, since $k_1 < k_2$, it must be:

$$\begin{aligned} & w\left(\kappa\left(\pi'\right)\right)-w\left(\kappa\left(\pi\right)\right)+\left[f'\left(\kappa\left(\pi'\right)\right)-f'\left(\kappa\left(\pi\right)\right)\right]k_2 \\ & +\left[C_1\left(\pi_1\right)-C_1\left(\pi_1'\right)+C_2\left(\pi_2\right)-C_2\left(\pi_2'\right)\right]k_2/K \\ > \; & 0 \end{aligned}$$

which means:

$$\begin{aligned} & w\left(\kappa\left(\pi'\right)\right)+f'\left(\kappa\left(\pi'\right)\right)k_2-\left[C_1\left(\pi_1'\right)+C_2\left(\pi_2'\right)\right]k_2/K \\ > \; & w\left(\kappa\left(\pi\right)\right)+f'\left(\kappa\left(\pi\right)\right)k_2-\left[C_1\left(\pi_1\right)+C_2\left(\pi_2\right)\right]k_2/K \end{aligned}$$

that is the RHS of (4.15). ■

Proof of Proposition 7. The optimal solution is inside the unit square $(0,1)\times(0,1)$ or belongs to the frontier: $\pi^*_{ik} = 0$ or $\pi^*_{ik} = 1$ for some i.

We want to prove that either is interior or is equal to $\pi^*_k = (1, 1)$.

We first exclude the possibility $\pi^*_{ik} = 0$. Indeed, according to equation (4.17), $\lim_{\pi_i \to 0^+} \partial\sigma_k/\partial\pi_i = +\infty$ and σ_k increases with π_i for $i = 1, 2$. Assume now $\pi^*_{ik} = 1$ for some i. If the solution exists and it is neither $\pi^*_{jk} = 0$ nor $\pi^*_{jk} = 1$, then $\pi^*_{jk} \in (0, 1)$, for which $\partial\sigma_k/\partial\pi_j = 0$. Notice that σ_k is smooth. Evaluating the second partial derivative in such solution, under Assumption 6, we get:

$$\frac{\partial^2\sigma_k}{\partial\pi_j^2} = -\frac{C'_j(\pi_j)}{\pi_j}\frac{k}{K}\left[\gamma_j + \nu_j(2 + \varphi - \xi)\right] < 0$$

Then, there are no local partial minima; also, since $C'_j(1) = 0$ (see Assumption 3), σ_k is increasing in π_j and, under the restriction $\pi_i = 1$, it generically attains a local maximum at $(1, 1)$. Then the set of optimal solutions is included in a positive-sloped (possibly disconnected) locus in the π-plane; according to Lemma 5:

$$\{\pi^*_k\} \subseteq \{\pi_k : \partial\sigma_k/\partial\pi_i = 0, i = 1, 2\} \cup \{(1, 1)\}$$

Eventually, we need to prove that $k_2 > k_1$ implies $\pi^*_{k_2} > \pi^*_{k_1}$. Assume at the contrary that k_1 and k_2 are such that $k_1 < k_2$ and $\pi^*_{k_2} < \pi^*_{k_1}$. Then, according to Lemma 6 and implication (4.14), $\sigma_{k_2}\left(\pi^*_{k_2}\right) > \sigma_{k_2}\left(\pi^*_{k_1}\right)$ entails that $\sigma_{k_1}\left(\pi^*_{k_2}\right) > \sigma_{k_1}\left(\pi^*_{k_1}\right)$, which contradicts the fact that $\pi^*_{k_1}$ is the best choice for the individual endowed with k_1. ■

Chapter 5

Optimal immigration policy when the public good is rival

5.1 Introduction

"When a man migrates from the one country to another, he abandons his share of public property [..] in the former country and acquires a share of public property in the latter [..] imposing a cost upon the original residents of the country to which he goes [..; however] the"public property" effect is contrasted with other sources of costs and benefits of migration" (Usher, 1977).

The economic effects of migration flows into developed economies have been widely analyzed by a rich literature.[1] Indeed, migration inflows are either beneficial or harmful dependently upon manifold contingent features of both host countries and immigration flows. As Nannestad (2006) suggests, one of the main issues concerns the composition of migration flows. Clearly, in presence of adverse-selection effects, migration flows are not welfare improving.[2] Secondly, the economic impact of immigrants depends on the extent and the features of their absorption into the labor market.

[1] See Borjas (1994), for a general discussion on the welfare implications deriving from immigration. See Nannestad (2006), for a focus on the welfare implications of immigration when destination countries are welfare states.

[2] In the growing literature on adverse selection mechanisms, the "welfare magnets hypothesis" (Borjas 1999) deserves consideration. According to this hypothesis, generous welfare states could act as magnets for "bad" immigrants.

The neoclassical theory suggests that destination countries gain from immigration flows of workers as long as the physical capital endowment of natives differs from the one of immigrants. In particular, according to Kemniz (2001), immigration inflows benefit all natives if and only if the average immigrant owns more physical capital than the average native.

However, the distribution of the benefits is not necessarily symmetric amongst natives. If immigrants are totally absorbed in the workforce, wages in the destination country tend to decrease, while production is enhanced. The difference between the following increase in output and the share of output earned by immigrant workers is the "immigration surplus". Notice that these benefits must not be overestimated. According to Borjas (1999a), the "immigration surplus" is conditional upon the fact that at least some natives see their wage decrease compared to the pre-immigration situation.

Another crucial factor for evaluating the impact of migration flows into destination countries is the time perspective. Most of the existing literature evaluating welfare effects of immigration tends to see the phenomenon as a static one; therefore, the main focus is on a static comparison between pre and post-immigration welfare in the country. In a neoclassical growth framework of overlapping dynasties, Ben-Gad (2004) shows that both the benefits and the decrease of wages are much smaller; his elegant analysis demonstrates that static models that do not incorporate endogenous capital accumulation, elastic labor supply and transition paths, do overestimate the impact of immigration on both the distribution of income and the welfare of natives.

One of the most controversial issues of debate in Western countries concerns the economic drain of immigrants on welfare-states social programs. While there is a general agreement on the fact that, if immigrants are not absorbed in host-countries labor force, they are a burden on natives, little agreement has been found on the effects of perfectly absorbed and substitutable inflows of workers. In his article, Wildasin (1994) provides a characterization of the income distribution frontier in host countries both in presence and in absence of migration inflows; his static analysis shows that, if immigrants are beneficiaries of income transfers, migration can lead to Pareto-inferior outcomes in all destination regions. Michael and Hatzipanayotou (2001) provide a more detailed focus on welfare policies; their analysis shows that the effect of immigrants crucially depends on public good provision and on how welfare policies are financed. When consumption tax revenues finance the provision of a public good, marginal migration increases natives' welfare in destination countries; when the consumption tax is equally redistributed, marginal migration decreases the welfare of natives. More ambiguous outcomes arise from tariffs revenue. However, their static analysis provides a one-shot picture and does not allow for long term considerations. Using a similar framework in presence of internationally mobile capital, Michael (2003) shows that income taxes and transfers can improve the welfare of natives even if immigrants provide smaller tax revenues to the state; the hypothesis of international capital mobility is thus crucial. The key role of capital is also stressed by Usher (1977); while his back-to-envelope calculations suggest that immigrants cannot but be a drain on welfare states, he stresses the importance of a long-term perspective.

In light of the above considerations, we focus[3] on long-term optimal immigration policies in welfare states. We consider a dynamic general equilibrium framework with capital accumulation; we assume the perspective of a policy maker who aims at maximizing the welfare of his native population. Her/his optimization program takes into account the arrival of foreign workforce: its impact on natives' incomes and on natives' public-good consumption. As in Michael (2003), we suppose that the government provides his citizens with a rival but non-excludible public good;[4] marginal immigration has thus a negative effect on natives' public-good utility. This effect is however offset by the contribution of immigrants (who are also allowed for a degree of free riding) to government revenues. The determination of the optimal immigration policy is in fact strictly linked to the determination of the optimal fiscal policy.

Optimal fiscal and immigration policies result from the incorporation of natives' utility maximization programs into the government welfare function; immigrants' optimal decisions on saving and consumption are also considered. Notice in particular that we account also for natives' attitudes towards migration. Indeed, the recent political debate in Europe shows that even if immigration flows have been for most countries "[...] a boon for their economy, [...] hostility to immigration is becoming a mainstream [...]" (*The Economist*, 2008). In support of this evidence, O'Rourke and Sinnot (2006) and Gang, Rivera-Batiz and Yun (2002) stress that natives' preferences towards immigration are significantly affected by nationalistic sentiments.

[3]This Chapter has been submitted to the *Journal of Public Economic Theory* as "Optimal immigration policy when the public good is rival" in a shorter version (with S. Bosi and H. Jayet).

[4]Think for instance of public health and education: too many recipients of these services can negatively affect the quality of the service (congestion).

On the other hand, Bisin and Verdier (2000) and Bisin, Topa and Verdier (2001) remark the significance of immigrants' assimilation resilience. These considerations suggest that the impact of foreign workforce on host countries goes beyond issues related to income distribution; immigration inflows represent a cultural challenge for host countries' natives and, in this light, a disutility[5]. In order to account for the disutility related to the cultural challenge, we introduce cultural heterogeneity costs in natives' utility function.

Our analysis aims at contributing to the actual debate on optimal immigration policies in the Western world.[6] It allows us to characterize optimal immigration and fiscal policies and the dynamics following possible shocks in policy parameters. It shows that even if policy-makers optimize the welfare of natives only and account for both the effects of congestion (of public goods) and of hostility towards immigrants, it is optimal to let (at least some) immigrants enter the country. The analysis also shows that, in absence of costs of cultural heterogeneity, it is optimal for natives to let borders open; indeed, the negative effect deriving from the congestion of the public good is more than offset by the positive effects related to factor complementarity.

In order to account for the possibility of costs of cultural heterogeneity, we focus on homogeneous functional forms; costs of cultural heterogeneity lead us to characterize an interior solution for frontier openness. We also show that an increase in the degree of intolerance has a negative effect on the optimal number of immigrants, but, surprisingly, no effects on the optimal fiscal policy. The impact of an increase in free riding and taxation on the optimal immigration policy is not univocal and depends on the degree of substitutability between the public and the private good in natives' consumption: given the stronger contribution of immigrants to the provision of the private good, the higher substitutability, the stronger the incentive to let immigrants come in (even if the provision of the public good diminishes). Conversely, an increase in the number of immigrants does not affect the optimal taxation.

The Chapter is organized as follows. In the next section, we introduce the model, while in the 5.3 we define the intertemporal equilibrium. Section 5.4 is devoted to the optimal fiscal and immigration policies. Short-run dynamics, namely cycles and monotonic convergence, are studied in sections 5.5 and 5.6. Concluding remarks are provided in section 5.7, while

[5] Alternatively, according to Nannestad (2004), integration can be seen as a public good.

[6] Throughout the analysis, our assumptions will be based on a relatively conservative stand. Nonetheless, immigration results to be optimal.

computational details are gathered in the Appendix.

5.2 The model

In this essay, we focus our attention on a Western welfare state. The country is populated by n_1 identical infinite-lived natives and n_2 infinite-lived immigrants. Natives are endowed with human and physical capital, but supply no labor.[7] When immigrants enter the country they do not dispose of (human or physical) capital; however, once arrived in the destination country, they can choose to accumulate capital. Conversely, natives always supply a positive quantity of human and physical capital; we also suppose that natives are more patient than immigrants, i.e., they have a higher discount rate. Immigrants supply inelastically one unit of labor.

Both capital and labor are employed in the production of a privately provided consumption good; an imperfect public good, which is non-excludible but rival, is also provided and consumed.

We assume that the policy maker of the country is benevolent toward her/his native population; s/he maximizes a domestic welfare function that only accounts for natives' utility. S/he chooses the fiscal pressure (and thus, the amount of public good) and the number of immigrants to let enter the country.

In the following, all variables referring to natives are labelled with 1; all variables referring to immigrants are labeled with 2. In addition, k_i, c_i and g_i (referred to the ith agent) denote respectively the amount of capital, the consumption good and the public good. These goods are produced with the same technology and, therefore, have the same price (normalized to one for simplicity). Finally, β_i and u_i will denote the discount factor and the utility function, respectively.

[7]It is reasonable to assume that the native population of Western developed countries is relatively more endowed of physical and human capital. Here we could alternatively suppose that natives do also supply inelastically one unit of labor. As long as the native's capital endowment is significantly larger than one, this wouldn't affect qualitative results of the analysis.

5.2.1 Firms

Suppose that in every period a competitive (representative) firm maximizes the profit $F(K,L) - rK - wL$ taking factor prices (the interest rate r and the wage rate w) and technology as given. The constant return to scale production function $F(K,L)$ satisfies additional neoclassical features. Let $f(k) \equiv F(k,1)$, where $k \equiv K/L$ is capital intensity.

Assumption 1 *(technology) F is homogeneous of degree 1 with $F \in C^2$, $F_K > 0$, $F_L > 0$, $F_{KK}F_{LL} > F_{KL}^2$ for every (K,L) such that $K, L > 0$.*

Assumption 2 *(Inada). Moreover, $f(0) = 0$, $\lim_{k\to 0} f'(k) = +\infty$ and $\lim_{k\to +\infty} f'(k) = 0$.*

Under Assumption 1, profit maximization requires

$$r = f'(k) \equiv r(k) \tag{5.1}$$

$$w = f(k) - kf'(k) \equiv w(k) \tag{5.2}$$

Second order restrictions for profit maximization are also satisfied under Assumption 1.

The following elasticities summarize the technological features and will play a role in the sequel:

$$\varepsilon_r(k) \equiv \frac{r'(k)k}{r(k)} = -\frac{1-s(k)}{\sigma(k)} < 0 \tag{5.3}$$

$$\varepsilon_w(k) \equiv \frac{w'(k)k}{w} = \frac{s(k)}{\sigma(k)} > 0$$

where $s(k) \equiv f'(k)k/f(k) \in (0,1)$ is the capital share in total income and $\sigma(k) \equiv [s(k)-1] f'(k) / [kf''(k)] > 0$ is the elasticity of capital-labor substitution.

When s is constant, the technology is Cobb-Douglas. When σ is constant, the technology is CES (constant elasticity of substitution).

In the case of a Cobb-Douglas technology:

$$F(K,L) \equiv AK^s L^{1-s} \tag{5.4}$$

where $s \in (0,1)$ is the (constant) capital share in total income, and $f'(k) = sAk^{s-1}$.

In the case of a CES technology:

$$F(K,L) \equiv A\left[\varkappa K^{\frac{\sigma-1}{\sigma}} + (1-\varkappa)L^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{5.5}$$

where $\sigma \geq 0$ is the (constant) elasticity of capital-labor substitution, and

$$f'(k) = \varkappa A\left[\varkappa + (1-\varkappa)k^{\frac{1-\sigma}{\sigma}}\right]^{\frac{1}{\sigma-1}}$$

The capital share depends now on the capital intensity:

$$s(k) = \frac{\varkappa}{\varkappa + (1-\varkappa)k^{\frac{1-\sigma}{\sigma}}} \tag{5.6}$$

The Cobb-Douglas technology satisfies Assumption 2, while the CES one does not.

5.2.2 Consumers

We focus now the attention on the inhabitants of the host country. The country is populated both by natives and immigrants; both types of agents choose their optimal consumption levels, given their own budget constraint. Notice in particular that agents' utility depends on consumption levels of both the consumption good – that is produced by firms – and on the public good – provided by the government.

Consider first the perspective of native citizens. Their utility function positively depends on the consumption good (c_{1t}) and on public good levels (g_{1t}). In order to proxy for the disutility deriving from the cultural challenge associated to the arrival of newcomers, we have introduced costs of cultural heterogeneity into their utility function. We assume that this disutility is a positive function of the number of foreigners (n_2) in total population ($n_1 + n_2$). Indeed, when in 1913 this ratio reached its historical peak (15%) in US, hostility to immigration pushed the population to demand (and eventually obtain) tough limits on migration that slowed the inflow for decades (*The Economist*, 2008). The evidence of Canada and US southern states shows also that for high levels of the ratio foreigners/total population, the national language of a country can also be challenged: the higher the ratio, the tougher the cultural challenge.

Finally, in order to account for the evolution of hostility over time, we discount cultural heterogeneity costs by a factor $\mu < 1/\beta_1$.

Natives maximize the following utility function:

$$\max_{k_{1t},c_{1t}} \sum_{t=0}^{\infty} \beta_1^t \left[u_1\left(c_{1t}, g_{1t}\right) - \mu^t v_1 \left(\frac{n_2}{n_1 + n_2} \right) \right]$$

where v_1 captures subjective costs of cultural heterogeneity. Diversity costs are a positive function of the number of newcomers but decrease during time as soon as immigrants gradually mix with the native population. Notice however, that this does not necessarily imply that immigrants are assimilated over time (*i.e.* converge) to natives. Indeed, while recent empirical evidence on the state of New York shows that hostility is only addressed towards recent immigration inflows, the melting-pot assimilation pattern has proved wrong (Bisin and Verdier, 2000). More generally, the case of an increasing negative bias towards immigrants is captured by $\mu > 1$, while $\mu < 1/\beta_1$ ensures the convergence of the series.

Natives are subject to the following budget constraint:

$$c_{1t} + k_{1t+1} - (1-\delta)\, k_{1t} \leq (1-\tau)\, r_t k_{1t}$$

where τ is the constant tax rate and δ is the depreciation rate, with $k_{10} > 0$. We assume therefore that natives have positive initial endowments of capital. Indeed, capital should be here interpreted as physical, financial but also as human capital. Therefore, starting from the first period, natives do supply a positive quantity of (physical and human) capital.

We now focus the attention on the latecomer population. We suppose that all immigrants have access to both the consumption good and the public good;[8] however, in order to capture phenomena such as clandestine immigration and black labor-market networks, we suppose that immigrants contribute only partially to government revenues. Clearly, if latecomers can enjoy the provision of public goods without paying for them, this creates a potential free-riding problem for welfare states. Schultz and Sjöström (2001) propose the accumulation of public debt as a possible tool to mitigate the free-rider problem. Indeed, if the public good is financed

[8] Myers and Papageorgiou (2000) show that, if illegal immigrants can be excluded from the redistribution of public services, it is optimal for the host country to impose a zero quota for legal immigration and spending nothing to enforce it, letting therefore illegal immigrants enter the country.

by local debt, immigrants will need to pay it by sharing the debt burden. Their two-periods analysis shows that in equilibrium there will be either too much debt and too little public good; this policy recommendation is far from being optimal.

The utility of immigrants positively depends on the consumption good, c_{2t}, and on the public good, g_{2t}, *i.e.*, they maximize the following utility function:

$$\max_{k_{2t},c_{2t}} \sum_{t=0}^{\infty} \beta_2^t u_2 \left(c_{2t}, g_{2t}\right)$$

Their optimal decisions are subject to the following constraint:

$$c_{2t} + k_{2t+1} - (1-\delta) k_{2t} \leq (1-\pi\tau)\left(r_t k_{2t} + w_t\right)$$

with $k_{20} = 0$, implying that when immigrants enter the country they have no capital. However, the constraint does not exclude *a priori* the possibility for them to accumulate capital.

In order to capture immigrants' free riding, we introduce the degree of participation to government's public expenditure, $\pi \in [0,1]$ or, equivalently, the free-riding degree, $1-\pi$. Clearly, one of these possibilities may arise: (1) $\pi = 1$ (citizens and immigrants pay the same tax rate), (2) $\pi < 1$ (free riding).

The following assumptions define agents' preferences, impatience and immigrants' free riding.

Assumption 3 *(preferences)* $u_i \in C^2$, $u_{ic} > 0$, $u_{ig} > 0$, $u_{icc} < 0$, $u_{icc}u_{igg} > u_{icg}^2$ *and* $\lim_{c_i \to 0} u_{ic}(c_i, g_i) = \lim_{g_i \to 0} u_{ig}(c_i, g_i) = +\infty$ *for every* (c_i, g_i) *with* $c_i, g_i > 0$. *In addition,* $v_1 \in C^2$, $v_1 > 0$, $v_1' > 0$, $v_1'' > 0$ *for every* $n_2 > 0$.

Assumption 4 *(discounting)* $\beta_1, \beta_2 \in (0,1)$, $\mu \in (0, 1/\beta_1)$ *and*

$$\beta_2 < \left[1 - \delta + \frac{1-\pi\tau}{1-\tau}\left(\frac{1}{\beta_1} - 1 + \delta\right)\right]^{-1} \tag{5.7}$$

Condition (5.7) is a crucial assumption of the model and deserves some comments.

(1) It generalizes the basic restriction in Becker (1980): indeed, the original assumption $\beta_2 < \beta_1$ is recovered, by setting $\pi = 1$ (no free riding).

(2) It imposes a restriction on free riding, given β_1, β_2. Indeed, we can rewrite inequality (5.7) in terms of π, *i.e.*:

$$\pi > \frac{1}{\tau}\left[1 - (1-\tau)\frac{1/\beta_2 - 1 + \delta}{1/\beta_1 - 1 + \delta}\right] \equiv \pi^*$$

(3) It implies discounting heterogeneity. Indeed $\pi^* < 1$ implies $\beta_2 < \beta_1$.

Immigrants' impatience (Assumption 4) implies that at the steady state (and in a neighborhood) immigrants will hold no capital.

Assumption 5 *(participation degree)* $\pi > \max\{\pi^*, 1 - \sigma/(1-s)\}$.

Assumption 5 reduces to $\pi > \pi^*$ if inputs are sufficiently substitutable ($\sigma > 1 - s$).

Notice finally that the above assumptions have important implications for immigrants' capital endowments as shown in the following proposition.

Proposition 10 *Under Assumptions 1-4, if k_{10} is sufficiently close to the steady state value (expression (5.20) below), then $k_{2t} = 0$, for every $t \geq 0$.*

Proof. The basic proof is in Becker (1980) and Becker and Foias (1987, 1994). For a formal proof in a model with elastic labor supply, see Bosi and Seegmuller (2008). Due to the complexity of our model, in order to ensure the existence of a "dominant consumer", we need to generalize the basic restriction $\beta_2 < \beta_1$. Comparing the Euler equation of the patient agent, *i.e.*:

$$\frac{u_{1c}(c_{1t}, g_{1t})}{u_{1c}(c_{1t+1}, g_{1t+1})} = \beta_1 [1 - \delta + (1-\tau) r_{t+1}] \tag{5.8}$$

with that of the impatient agent:

$$\frac{u_{2c}(c_{2t}, g_{2t})}{u_{2c}(c_{2t+1}, g_{2t+1})} = \beta_2 [1 - \delta + (1-\pi\tau) r_{t+1}]$$

we require at the stationary equilibrium:

$$\beta_2 [1 - \delta + (1-\pi\tau) r] < \beta_1 [1 - \delta + (1-\tau) r] = 1$$

Solving for r the equation on the right-hand side and substituting r in the inequality on the left-hand side, we get the inequality (5.7) in Assumption 4. ■

In the following, we will focus only on equilibria around the steady state. In this case, Proposition 10 simplifies the immigrants' optimization program. It implies that, in every period, immigrants consume only and completely their labor income, *i.e.*:

$$c_{2t} = (1 - \pi\tau)\, w_t \tag{5.9}$$

We thus now shift the attention towards the optimization problem of the native population. Under Assumption 3 and 4, natives' optimization program entails an Euler equation:

$$\frac{u_{1c}\,(c_{1t}, g_{1t})}{u_{1c}\,(c_{1t+1}, g_{1t+1})} = \beta_1 \left[1 - \delta + (1 - \tau)\, r_{t+1}\right]$$

a binding budget constraint:

$$c_{1t} + k_{1t+1} = \left[1 - \delta + (1 - \tau)\, r_t\right] k_{1t} \tag{5.10}$$

and a transversality condition:

$$\lim_{t \to +\infty} \beta_1^t u_{1c}\,(c_{1t}, g_{1t})\, k_{1t+1} = 0$$

which will be satisfied in a neighborhood of the steady state (since $\beta_1 < 1$).

Notice finally that the representative citizen's capital depends on the capital intensity as follows:

$$k_{1t} = k_t n_2 / n_1 \tag{5.11}$$

5.2.3 Government

Assume now the perspective of a benevolent government who chooses the optimal (fiscal and immigration) policy by taking into account the welfare of citizens only. Given that immigrants do not vote, we assume that the policy maker does not account for their preferences. Since the public good is rival but non-excludible, the individual amounts consumed by citizens and immigrants are the same, *i.e.*:

$$g_{1t} = g_{2t} = G_t / (n_1 + n_2)^{\rho} \tag{5.12}$$

where G is the aggregate public good, financed through the tax receipt:

$$G_t = \tau (n_1 r_t k_{1t} + \pi n_2 w_t) \tag{5.13}$$

and $\rho \in [0, 1]$ is the degree of rivality of the public good: according to Samuelson's definition, when $\rho = 0$ the public good is pure while when $\rho = 1$, it is fully rival.

Substituting (5.13) in (5.12) and using (5.11), we have

$$g_{1t} = g_{2t} = \tau (r_t k_t + \pi w_t) \frac{n_2}{(n_1 + n_2)^{\rho}} \tag{5.14}$$

Given the equilibrium prices $r_t(\tau, n_2)$ and $w_t(\tau, n_2)$, and the equilibrium quantities $k_t = k_t(\tau, n_2)$ and $c_{1t} = c_{1t}(\tau, n_2)$, the government maximizes a (domestic) welfare function

$$\begin{aligned}
& \omega(\tau, n_2) \\
\equiv\; & \sum_{t=0}^{\infty} \beta_1^t \left[u_1(c_{1t}, g_{1t}) - \mu^t v_1 \left(\frac{n_2}{n_1 + n_2} \right) \right] \\
=\; & \sum_{t=0}^{\infty} \beta_1^t u_1 \left(c_{1t}(\tau, n_2), \tau \left[r_t(\tau, n_2) k_t(\tau, n_2) + \pi w_t(\tau, n_2) \right] \frac{n_2}{(n_1 + n_2)^{\rho}} \right) \\
& - \sum_{t=0}^{\infty} (\mu \beta_1)^t v_1 \left(\frac{n_2}{n_1 + n_2} \right)
\end{aligned} \tag{5.15}$$

with respect to the fiscal policy τ and the immigration policy n_2.

5.3 Equilibrium

The mechanisms that lead the system to equilibrium are based on the following timing:

(1) The government commits itself to a policy-mix, *i.e.* to a fiscal pressure τ and to a number of immigrants n_2.

(2) Citizens and immigrants maximize their respective utility functions.

(3) Markets clear.

5.3.1 Equilibrium in the labor market

We have assumed for simplicity that natives provide both human and physical capital; on the other hand, immigrants provide labor. As mentioned above, as long as natives provide more human capital than labor, qualitative results wouldn't change if we allowed also natives to provide labor.

In equilibrium, total labor coincides thus with the number of immigrants, *i.e.*, $L_t = n_2$.

Notice finally that we have implicitly assumed that all n_2 individuals are willing to migrate from their source country. We suppose in practice that immigrants' wage in their source country is lower than the one they receive in the destination country (for simplicity, we assume that their reservation wage is equal to zero) and their labor supply is inelastic; therefore, they always find profitable to migrate.

5.3.2 Equilibrium in the good market

Aggregate demand is the sum of aggregate consumption, investment and public spending and, at each date, it is equal to aggregate production:

$$n_1 c_{1t} + n_2 c_{2t} + n_1 \left[k_{1t+1} - (1-\delta)\, k_{1t}\right] + G_t = n_2 f\left(k_t\right) \tag{5.16}$$

Substituting (5.9), (5.11), (5.13) in (5.16), we obtain

$$r_t k_t + w_t = f\left(k_t\right)$$

according to the homogeneity of the production function (Assumption 1). In other words, firm's technology, (5.1)-(5.2), and the lack of pure profit ensure the equilibrium in the good market.

5.3.3 Dynamic system

Substituting (5.1), (5.2), (5.11), (5.14) in the Euler equation (5.8) and natives' budget constraint (5.10) gives the following two-dimensional dynamic system in the pair (k_t, c_{1t}). k_t is a predetermined variable, while c_{1t} is a jump variable.

$$\frac{u_{1c}\left(c_{1t}, \tau\left[r\left(k_t\right)k_t + \pi w\left(k_t\right)\right]\frac{n_2}{(n_1+n_2)^{\rho}}\right)}{u_{1c}\left(c_{1t+1}, \tau\left[r\left(k_{t+1}\right)k_{t+1} + \pi w\left(k_{t+1}\right)\right]\frac{n_2}{(n_1+n_2)^{\rho}}\right)} = \beta_1\left[1-\delta+(1-\tau)\,r\left(k_{t+1}\right)\right] \tag{5.17}$$

$$\left[1-\delta+(1-\tau)\,r\left(k_t\right)\right]n_2 k_t = n_1 c_{1t} + n_2 k_{t+1} \tag{5.18}$$

5.3.4 Steady state

It is now possible to compute the steady state. The stationary interest rate is determined by the Euler equation (5.17):

$$r = \frac{1/\beta_1 - 1 + \delta}{1-\tau} \tag{5.19}$$

which gives also capital intensity:

$$k = r^{-1}\left(\frac{1/\beta_1 - 1 + \delta}{1-\tau}\right) \tag{5.20}$$

while (5.11) determines the individual capital level:

$$k_1 = \frac{n_2}{n_1}k \tag{5.21}$$

More explicitly, when the production function is a Cob-Douglas and a CES function, respectively, capital intensity becomes:

$$k = \left[\frac{sA\,(1-\tau)}{1/\beta_1 - 1 + \delta}\right]^{\frac{1}{1-s}} \tag{5.22}$$

$$k = \left[\frac{1}{1-\varkappa}\left(\left[\frac{\varkappa A\,(1-\tau)}{1/\beta_1 - 1 + \delta}\right]^{1-\sigma} - \varkappa\right)\right]^{\frac{\sigma}{1-\sigma}} \tag{5.23}$$

respectively.

Natives' budget constraint (5.18) determines their consumption:

$$c_1 = \frac{1-\beta_1}{\beta_1}\frac{n_2}{n_1}k \tag{5.24}$$

while the individual provision of public good is given by (5.14):

$$\begin{aligned} g_1 &= g_2 = \tau\left[r(k)k + \pi w(k)\right]\frac{n_2}{(n_1+n_2)^{\rho}} \\ &= \tau r k \frac{s+(1-s)\pi}{s}\frac{n_2}{(n_1+n_2)^{\rho}} \end{aligned} \tag{5.25}$$

The private/public good ratio is also a variable of interest:

$$\frac{c_1}{g_1} = \frac{1-\tau}{\tau}\frac{(n_1+n_2)^{\rho}}{n_1}\frac{1-\beta_1}{1-\beta_1(1-\delta)}\frac{s}{s+(1-s)\pi} \tag{5.26}$$

In order to obtain parametric solutions for k_1, c_1, g_1, simply replace (5.20) (or, more explicitly, (5.22) or (5.23)) in (5.21), (5.24), (5.25), respectively. Notice that neither aggregate capital nor the interest rate depend on the number of immigrants; to the contrary, the individual consumption levels of both the private and the public good are increasing functions of n_2.

5.3.5 Comparative statics

In order to understand more deeply the long-run effects of fiscal and immigration policies we now focus on their impact on the steady state; we thus compute the elasticities of steady-state values with respect to the policy parameters, (τ, n_2).[9] The following proposition characterizes the sign of these elasticities.

Proposition 11 *The elasticities of the steady state values with respect to the policy parameters*

[9] We will denote with $\varepsilon_{yx} \equiv (\partial y/\partial x)\, x/y$ the elasticity of y with respect to x.

have the following signs.

$$\begin{bmatrix} \varepsilon_{k\tau} & \varepsilon_{kn_2} \\ \varepsilon_{c_1\tau} & \varepsilon_{c_1n_2} \\ \varepsilon_{k_1\tau} & \varepsilon_{k_1n_2} \\ \varepsilon_{g_1\tau} & \varepsilon_{g_1n_2} \end{bmatrix} = \begin{bmatrix} -\frac{\tau}{1-\tau}\frac{\sigma}{1-s}\,(<0) & 0 \\ -\frac{\tau}{1-\tau}\frac{\sigma}{1-s}\,(<0) & 1 \\ -\frac{\tau}{1-\tau}\frac{\sigma}{1-s}\,(<0) & 1 \\ 1-\frac{\tau}{1-\tau}\frac{s}{1-s}\frac{s+(1-s)\pi+\sigma-1}{s+(1-s)\pi}\,\left(\lesseqgtr 0\right) & \frac{n_1+(1-\rho)n_2}{n_1+n_2}\,(\in(0,1]) \end{bmatrix} \tag{5.27}$$

Moreover, $\varepsilon_{g_1\tau} > 0$ *if and only if* $\tau < \bar{\tau}$*, where, under Assumption 4,*

$$\bar{\tau} \equiv (1-s)\frac{s+(1-s)\pi}{s\sigma+(1-s)\pi} \in (0,1) \tag{5.28}$$

Proof. We derive the elasticities in (5.27), by using (5.3) and noticing that the elasticity of the capital share with respect to capital is $ks'(k)/s(k) = -(1-s)(1-\sigma)/\sigma$. We observe that the signum of the elasticity of individual provision of public good with respect to the tax rate is positive ($\varepsilon_{g_1\tau} > 0$) if and only if $\tau < \bar{\tau}$. The right-hand side in (5.28) is always positive and, under Assumption 5, less than one. ■

We observe that the tax rate has a positive effect on the interest rate and a negative effect on the capital intensity according to (5.3), independently of the immigration policy.

5.4 Nationalist policy

Suppose now that the government's welfare function only accounts for the welfare of native citizens (see (5.15)); in practice, while deciding the fiscal and immigration policies, s/he does not account for the welfare of immigrants. Given that the government is elected by citizens only, this assumption can be considered reasonably realistic.

In order to compute analytically the government optimization program, we maximize the welfare function (5.15) evaluated at the steady state, with respect to the policy (τ, n_2), i.e.:

$$\omega(\tau, n_2) = \frac{1}{1-\beta_1}u_1(c_1, g_1) - \frac{1}{1-\mu\beta_1}v_1\left(\frac{n_2}{n_1+n_2}\right) \tag{5.29}$$

First, we observe that, at the steady state, k is a function of τ, but not of n_2 (see equation (5.20)): $k = k(\tau)$.

Second, we notice that c_1 and g_1 are indirect functions of (τ, n_2) as follows (see equations (5.24) and (5.25)):

$$
\begin{aligned}
c_1(\tau, n_2) &= \frac{1-\beta_1}{\beta_1}\frac{n_2}{n_1}k(\tau) \\
g_1(\tau, n_2) &= g_2 = \tau\left[r(k(\tau))k(\tau) + \pi w(k(\tau))\right]\frac{n_2}{(n_1+n_2)^{\rho}} \\
&= \tau r(k(\tau))k(\tau)\frac{s(k(\tau)) + (1-s(k(\tau)))\pi}{s(k(\tau))}\frac{n_2}{(n_1+n_2)^{\rho}} \\
\omega(\tau, n_2) &= \frac{1}{1-\beta_1}u_1(c_1(\tau, n_2), g_1(\tau, n_2)) - \frac{1}{1-\mu\beta_1}v_1\left(\frac{n_2}{n_1+n_2}\right)
\end{aligned}
$$

Finally, we compute the following gradients:

$$
\frac{\partial \omega}{\partial \tau} = \frac{1}{1-\beta_1}\left(\frac{\partial u_1}{\partial c_1}\frac{c_1}{\tau}\varepsilon_{c_1\tau} + \frac{\partial u_1}{\partial g_1}\frac{g_1}{\tau}\varepsilon_{g_1\tau}\right) \tag{5.30}
$$

$$
\frac{\partial \omega}{\partial n_2} = \frac{1}{1-\beta_1}\left(\frac{\partial u_1}{\partial c_1}\frac{c_1}{n_2}\varepsilon_{c_1n_2} + \frac{\partial u_1}{\partial g_1}\frac{g_1}{n_2}\varepsilon_{g_1n_2}\right) - \frac{n_1}{(n_1+n_2)^2}\frac{v_1'}{1-\mu\beta_1} \tag{5.31}
$$

where $\varepsilon_{c_1\tau}$, $\varepsilon_{c_1n_2}$, $\varepsilon_{g_1\tau}$, $\varepsilon_{g_1n_2}$ are given by (5.27).

We have assumed for simplicity that the government does not announce an optimal non-stationary policy $\{(\tau_t, n_{2t})\}_{t=0}^{\infty *}$ but commits itself to implement a constant policy over time.[10]

In the following, we first characterize the optimal fiscal policy for a given immigration policy; then, we compute the optimal immigration policy for a given fiscal policy. Finally, we will focus on the optimal policy mix under more explicit functional forms.

5.4.1 Optimal fiscal policy

Equations (5.30) and (5.31) allow us to focus on optimal fiscal and immigration policies, respectively. We first concentrate our attention on the optimal fiscal policy. As shown in the next proposition, equation (5.30) determines the optimal level of taxation, given an immigration

[10] Clearly, the solution $(\tau, n_2)^*$ is time-consistent at the steady state, i.e., if the economic system is at the steady state from the beginning. Indeed, the objective (5.29) is invariant over time (since (c_1, g_1) are invariant) and the optimal solution does not change over time.

However, when the initial condition is outside the steady state, one may wonder not only whether, in general, the optimal sequence computed at $t+1$: $\{(\tau_s', n_{2s}')\}_{s=t+1}^{\infty *}$, is the consistent continuation of the sequence announced at time t: $\{(\tau_s, n_{2s})\}_{s=t}^{\infty *}$, (that is $(\tau_s', n_{2s}') = (\tau_s, n_{2s})$ for every $s > t$), but also whether a constant policy announced at time t: $\{(\tau, n_2)\}_{s=t+1}^{\infty *}$, is consistent with the optimal sequence at time $t+1$: $\{(\tau', n_2')\}_{s=t+1}^{\infty *}$, (that is $(\tau', n_2') = (\tau, n_2)$).

policy.

Proposition 12 *Given the immigration policy n_2, the optimal fiscal pressure is given by*

$$\tau^* = \bar{\tau}\left(1+\bar{\tau}\frac{\sigma}{1-s}\frac{\varepsilon_{u_1c_1}}{\varepsilon_{u_1g_1}}\right)^{-1} \tag{5.32}$$

where $\bar{\tau}$ is given by (5.28), and $\varepsilon_{u_1c_1}$, $\varepsilon_{u_1g_1}$ are the elasticity of utility with respect to the private and the public good, respectively. Under Assumption 5, $\tau^* \in (0,1)$.

Proof. Substituting (5.27) in (5.30), we obtain: $\partial\omega/\partial\tau \geq 0$ iff $\tau \leq \tau^*$. Then the maximum of ω is global at τ^*. Moreover, Assumption 5 entails $\bar{\tau} \in (0,1)$ and, therefore, $\tau^* \in (0,1)$ since $\varepsilon_{u_1c_1}/\varepsilon_{u_1g_1} > 0$. ■

$\bar{\tau}$ is a critical value: indeed a taxation degree beyond $\bar{\tau}$ implies a negative effect (a reversal) on the individual provision of public good ($\varepsilon_{g_1\tau} < 0$: see Proposition 11).

In the following, in order to provide explicit policy recommendations, we focus on explicit functional forms.

Consider the case of a homogeneous utility function of degree $1-1/\eta$:

$$u_1(c_1,g_1) \equiv \frac{\varphi(c_1,g_1)^{1-1/\eta}}{1-1/\eta} \text{ if } \eta \neq 1;\ u_1(c_1,g_1) \equiv \ln\varphi(c_1,g_1) \text{ if } \eta = 1 \tag{5.33}$$

where φ is a positive function homogeneous of degree one, while u_1 is homogeneous of degree $1-1/\eta$. η can be seen as the elasticity of intertemporal substitution of a composite good φ.

The homogeneity of the utility (5.33) implies that the private consumption share in the composite good φ depends only on the ratio private/public good: $\alpha = \alpha(c_1/g_1)$, where r and c_1/g_1 are explicitly given by (5.19) and (5.26). Simple computations yield:

$$\varepsilon_{u_1c_1} = \alpha\left(\frac{c_1}{g_1}\right)\frac{\eta-1}{\eta} \tag{5.34}$$

$$\varepsilon_{u_1g_1} = \left[1-\alpha\left(\frac{c_1}{g_1}\right)\right]\frac{\eta-1}{\eta} \tag{5.35}$$

We obtain

$$\tau^* = \bar{\tau}\left[1+\bar{\tau}\frac{\sigma}{1-s}\frac{\alpha(c_1/g_1)}{1-\alpha(c_1/g_1)}\right]^{-1} \tag{5.36}$$

where $\alpha = c_1\varphi_c/\varphi$ is the private consumption share in the composite good, which is a function of the private/public good ratio (5.26).

In the case of CES preferences, we set

$$\varphi(c_1, g_1) \equiv \left[\alpha c_1^{1-1/\varsigma} + (1-\alpha) g_1^{1-1/\varsigma}\right]^{\varsigma/(\varsigma-1)} \tag{5.37}$$

where ς is the elasticity of substitution between private and public good, and the fiscal rule becomes:

$$\tau = \bar{\tau}(\tau)\left[1 + \bar{\tau}(\tau)\frac{\sigma}{1-s}\frac{\alpha}{1-\alpha}\left(\frac{c_1}{g_1}\right)^{1-1/\varsigma}\right]^{-1} \tag{5.38}$$

where c_1/g_1 is given by (5.26) and depends on (τ, n_2). $\bar{\tau}$ depends on $s(k)$ and k is a function of τ. Thus, (5.38) is an implicit function, which defines $\tau^* = \tau^*(n_2)$.

In the Cobb-Douglas case for technology and preferences, where (5.4) holds and

$$\varphi(c_1, g_1) \equiv c_1^{\alpha} g_1^{1-\alpha} \tag{5.39}$$

we have $\varsigma = 1$ and (5.38) simplifies to:

$$\tau^* = \bar{\tau}\left(1 + \bar{\tau}\frac{\sigma}{1-s}\frac{\alpha}{1-\alpha}\right)^{-1} \tag{5.40}$$

where now $\bar{\tau}$ is a constant and no longer depends on τ (indeed, the capital share s is a constant). In this case, the optimal fiscal pressure τ^* becomes constant and neither the frontier openness (number of immigrants n_2) nor the elasticity of intertemporal substitution η affect τ^*. A stronger preference for the public good $(1-\alpha)$ increases the optimal tax rate τ^* so as to finance the provision of public good.[11]

In the above proposition, we have individuated the parameters that affect optimal taxation. In order to understand the role of these parameters, in the following corollary we compute the elasticities of optimal taxation relatively to the fundamentals.

Corollary 13 *There is no impact of discounting cultural heterogeneity* (μ) *on optimal taxation:* $\varepsilon_{\tau^*\mu} = 0$*. Moreover, if* $\varepsilon_{u_1 c_1}/\varepsilon_{u_1 g_1}$ *is constant, the impact of the fundamental parameters on*

[11] In the Cobb-Douglas case, φ is a weighted geometric average and can be interpreted as a composite good.

optimal taxation is the following:

$$\begin{aligned} \varepsilon_{\tau^*\rho} &= 0 \\ \varepsilon_{\tau^*\pi} &< 0 \text{ iff } \sigma < 1 \end{aligned} \quad (5.41)$$

Proof. Notice that, in general, $\bar{\tau}$ and $\varepsilon_{u_1c_1}/\varepsilon_{u_1g_1}$ do not depend on μ. Moreover, if $\varepsilon_{u_1c_1}/\varepsilon_{u_1g_1}$ is constant, τ^* does not even depend on ρ. Indeed, we know that σ does not depend on (ρ, π) because k of modified golden rule does not depend on these parameters, and $\bar{\tau}$ does not depend on ρ. However, τ^* depends only on π through $\bar{\tau}$, according to the following elasticity:

$$\varepsilon_{\tau^*\pi} = -s\tau^* \frac{(1-\sigma)\pi}{[s + (1-s)\pi]^2}$$

■

In the following corollary, we focus on the impact of the capital share and the capital-labor substitution in the case of a CES technology. In general, these two elasticities are linked by capital intensity (and thus, also in the CES case).

Corollary 14 *In the case of a CES technology, if $\varepsilon_{u_1c_1}/\varepsilon_{u_1g_1}$ is constant,*

$$\varepsilon_{\tau^*s} = \frac{s}{1-s}\left(\tau^* \frac{(1-\sigma)\pi}{[s+(1-s)\pi]^2} - 1\right) \quad (5.42)$$

$$\varepsilon_{\tau^*\sigma} = \frac{\tau^* - \bar{\tau}}{\bar{\tau}} - \sigma\varepsilon_{\tau^*s} \ln \frac{r}{\varkappa A} \quad (5.43)$$

Therefore, $\varepsilon_{\tau^*s} < 0$ iff

$$\sigma > \frac{(1 - [s + (1-s)\pi])(1-s)\pi}{(1 - [s + (1-s)\pi])(1-s)\pi + [s + (1-s)\pi]^2 (1 + \varepsilon_{u_1c_1}/\varepsilon_{u_1g_1})} (< 1) \quad (5.44)$$

Moreover, under a sufficiently large capital accumulation and capital-labor substitution, the impact of σ on the optimal fiscal pressure is negative ($\varepsilon_{\tau^*\sigma} < 0$).

Proof. In the case of a CES technology, σ no longer depends on s. If $\varepsilon_{u_1c_1}/\varepsilon_{u_1g_1}$ is also constant, (5.42) holds. Substituting τ^* in (5.42) gives (5.44).

By definition, σ is constant (that is, when the endogenous parameter s moves, other parameters move too in order to keep σ constant). However, given the other parameters, $s(k)$ depends on σ (because k does); according to (5.23), the steady-state capital share $s(k)$ is:

$$s(k) = \tilde{s}(\sigma) = \hat{s}(\tau) \equiv \varkappa \left[\frac{1/\beta_1 - 1 + \delta}{\varkappa A (1-\tau)} \right]^{1-\sigma} \tag{5.45}$$

The elasticity of $\tilde{s}$ is given by $\varepsilon_{\tilde{s}\sigma} = -\sigma \ln [r/(\varkappa A)]$. Tedious computations give also $\varepsilon_{\tau^*\sigma} = (\tau^* - \bar{\tau})/\bar{\tau} + \varepsilon_{\tau^* s}\varepsilon_{\tilde{s}\sigma}$ and, finally, (5.43). Since $(\tau^* - \bar{\tau})/\bar{\tau} < 0$, $\varepsilon_{\tau^* s} < 0$ under a sufficiently large capital-labor substitution (see (5.44)), and $r < \varkappa A$ under a sufficiently large capital accumulation; thus, the right-hand side in (5.43) is negative. ■

Our results show that the hostility discount factor μ has no impact on the optimal taxation rate because of the additively separable form. If the ratio $\varepsilon_{u_1 c_1}/\varepsilon_{u_1 g_1}$ is constant, also the degree of impureness of the public good ρ has no impact on the optimal taxation rate. Indeed, while the utility deriving from the public good to each agent depends also on ρ, her/his fiscal contribution is independent from it.

Moreover, in the case of a CES technology and a constant ratio $\varepsilon_{u_1 c_1}/\varepsilon_{u_1 g_1}$, under sufficiently high capital-labor substitution (inequality (5.44)), a higher capital share in total income implies a larger fiscal burden for natives. It follows that the optimal taxation rate will be lower. Indeed, since the capital-labor substitution magnifies the negative impact of taxation on natives' consumption and capital (see (5.27)), a higher σ entails the willing of lowering the fiscal pressure.

Finally, in the case of Cobb-Douglas preferences, we notice that the number of immigrants n_2 has no impact on the optimal fiscal pressure τ^*. However, inputs substitutability ($\sigma > 1$) implies a positive effect of immigrants' fiscal participation π on the optimal fiscal pressure τ^*. This somewhat surprising result can be explained as follows; at the margin, native's higher utility due to an increased provision of public good more than offsets the disutility arising from a heavier taxation.

5.4.2 Optimal immigration policy

We now shift our attention to the optimal immigration policy under a given fiscal policy. We will first consider the case of no costs of cultural heterogeneity and determine a general solution for the optimal policy. In order to characterize the general solution in presence of costs of cultural heterogeneity, we will then need to refer to explicit utility and cost functions.

Without costs of cultural heterogeneity

In this section we aim to disentangle the impact of public-good congestion and immigrants free-riding on natives' welfare. In particular, we question whether the congestion of public good coupled with an undesirable free riding in immigrants' fiscal participation actually represents a sufficient economic argument to limit the openness of frontiers and to introduce immigration quotas.

Our results show that, in absence of costs of cultural heterogeneity, the optimal immigration policy consists in opening borders. Surprisingly, this result holds for any degree ρ of impureness of the public good. Indeed, natives' gains coming from immigrants' labor force more than offset the disutility deriving from the congestion of the public good. This finding is far from trivial and is formalized in the following proposition; it can indeed be considered one of the main contributions of our work.

Proposition 15 *With no costs of cultural heterogeneity ($v_1 = 0$), $\partial\omega/\partial n_2 > 0$.*

Proof. According to (5.27) and (5.31), we have

$$\frac{\partial \omega}{\partial n_2} = \frac{1}{1-\beta_1}\left[\frac{\partial u_1}{\partial c_1}\frac{c_1}{n_2} + \frac{\partial u_1}{\partial g_1}\frac{g_1}{n_2}\frac{n_1 + (1-\rho)\,n_2}{n_1+n_2}\right] > 0 \tag{5.46}$$

because, under Assumptions 3, marginal utilities are strictly positive. ■

Notice that, for a given τ, neither the stationary state k nor r depend on n_2 (see (5.19)). However, the individual wealth $k_1 = kn_2/n_1$, current and future consumption $c_1 = [(1-\tau)\,r - \delta]\,kn_2/n_1$ and provision of public good $g_1 = \tau\,(rk + \pi w)\,n_2/\,(n_1+n_2)^{\rho}$ are all positive functions of n_2. Thus, the presence of immigrants has a positive impact on welfare.

Therefore, if citizens are not racist (no costs of cultural heterogeneity) but the government only considers natives' welfare (neglecting the welfare of immigrants) and the public good is rival (congestion costs), the optimal policy consists in completely opening frontiers. A different conclusion holds if people are racist: in this case, the degree of openness will depend on the degree of intolerance.

Finally, we also observe that $\partial\omega/\partial n_2$ decreases with ρ. This allows us to provide an additional (somehow obvious) result: the purer the public good (that is, the smaller the congestion costs), the stronger the positive impact of migrants on natives' welfare.

With costs of cultural heterogeneity

In presence of costs of cultural heterogeneity, (5.46) needs to be generalized so as to account for hostility towards immigrants. The welfare function to maximize with respect to n_2 is now represented by (5.29): the optimal immigration policy solves $\partial\omega/\partial n_2 = 0$.

The optimal immigration policy is thus the result of an arbitrage between the above benefits deriving from factor complementarity and the costs associated to hostility. In order to provide the explicit arbitrage equation we consider specific functional forms. In the case of the homogenous utility function (5.33), the arbitrage equation becomes (as shown in the Appendix):

$$\frac{u_1}{1-\beta_1}\frac{\eta-1}{\eta}\left(1-\rho\frac{n_2}{n_1+n_2}\left[1-\alpha\left(\frac{c_1}{g_1}\right)\right]\right)=\frac{v_1}{1-\mu\beta_1}\frac{n_1}{n_1+n_2}\varepsilon_{v_1} \tag{5.47}$$

where $\varepsilon_{v_1}(x)\equiv xv_1'(x)/v_1(x)$ is the elasticity of the costs of cultural heterogeneity with respect to $x\equiv n_2/(n_1+n_2)$. When the cost of cultural heterogeneity assume the following form:

$$v_1\left(\frac{n_2}{n_1+n_2}\right)\equiv\gamma\left(\frac{n_2}{n_1+n_2}\right)^{\theta} \tag{5.48}$$

we obtain $\varepsilon_{v_1}=\theta$, which no longer depends on the density of immigrants. γ can be interpreted as a degree of intolerance. $\theta\geq 1$ captures the increasing marginal costs of cultural heterogeneity.

In the case of CES technology and preferences (see (5.5) and (5.37), respectively), under definition (5.48), the immigration arbitrage (5.47) reduces to:

$$\frac{1-\alpha+\alpha\left(\frac{c_1}{g_1}\right)^{\frac{\varsigma-1}{\varsigma}}-(1-\alpha)\frac{\rho n_2}{n_1+n_2}}{(1-\beta_1)^{\frac{1}{\eta}}\left(\frac{n_1}{n_2}\frac{\beta_1}{k}\frac{c_1}{g_1}\right)^{1-\frac{1}{\eta}}\left[1-\alpha+\alpha\left(\frac{c_1}{g_1}\right)^{\frac{\varsigma-1}{\varsigma}}\right]^{\frac{\eta-\varsigma}{\eta(1-\varsigma)}}}=\frac{\gamma\theta}{1-\mu\beta_1}\frac{n_1}{n_2}\left(\frac{n_2}{n_1+n_2}\right)^{1+\theta} \tag{5.49}$$

where k and c_1/g_1 are explicitly given by (5.23) and (5.26), respectively (see the Appendix).

(5.49) is an explicit equation that can be easily used for numerical analysis; it allows to study the impact of each parameter on the optimal policy. In order to provide an analytical focus on the impact of the fundamentals on the optimal immigration policy we now provide an application based on Cobb-Douglas functions for technology and preferences (see (5.4) and (5.39)).[12] These functional forms allow us to simplify some parameters of the model; indeed, under the production function (5.4), the capital share s in total income is constant. Under the utility function (5.39), the consumption share α in the composite good is constant as well and no longer depends on the ratio c_1/g_1.

In this case, (5.47) reduces to:

$$\frac{1-(1-\alpha)\frac{\rho n_2}{n_1+n_2}}{(1-\beta_1)^{\frac{1}{\eta}}\left[\frac{n_1}{n_2}\frac{\beta_1}{k}\left(\frac{c_1}{g_1}\right)^{1-\alpha}\right]^{1-\frac{1}{\eta}}}=\frac{\gamma\theta}{1-\mu\beta_1}\frac{n_1}{n_2}\left(\frac{n_2}{n_1+n_2}\right)^{1+\theta} \tag{5.50}$$

(see the Appendix), where k and c_1/g_1 are explicitly given by (5.22) and (5.26), respectively.

By applying the Implicit Function Theorem to (5.50), we find the elasticities of n_2^* (optimal immigration policy, given the fiscal policy) with respect to the fiscal policy (τ) and to other structural parameters of interest ($\alpha, \mu, \gamma, \pi, \rho$) (see the Appendix), *i.e.*:

$$\begin{aligned}
\varepsilon_{n_2^*\gamma} &= -Q && (5.51)\\
\varepsilon_{n_2^*\mu} &= -Q\frac{\mu\beta_1}{1-\mu\beta_1} && (5.52)\\
\varepsilon_{n_2^*\pi} &= Q\frac{\eta-1}{\eta}(1-\alpha)\frac{(1-s)\pi}{s+(1-s)\pi} && (5.53)\\
\varepsilon_{n_2^*\tau} &= Q\frac{\eta-1}{\eta}\frac{\tau}{1-\tau}\left(\frac{1-\alpha}{\tau}-\frac{1}{1-s}\right) && (5.54)\\
\varepsilon_{n_2^*\rho} &= -Q\left[T+\frac{\eta-1}{\eta}(1-\alpha)\ln(n_1+n_2)^{\rho}\right] && (5.55)\\
\varepsilon_{n_2^*\alpha} &= Q\left[T+\frac{\eta-1}{\eta}(1-\alpha)\ln\frac{c_1}{g_1}\right]\frac{\alpha}{1-\alpha} && (5.56)
\end{aligned}$$

[12]The case $\sigma=\varsigma=1$ in terms of marginal rates of substitution.

where

$$\begin{aligned}
Q &\equiv \left[\left(1+\theta+\rho\left(1-\alpha\right)\frac{1-\eta}{\eta}\right)\left(\nu-\frac{n_2}{n_1+n_2}\right)+T\frac{n_1}{n_1+n_2}\right]^{-1} \\
\nu &\equiv \frac{\theta+\frac{1-\eta}{\eta}}{1+\theta+\rho\left(1-\alpha\right)\frac{1-\eta}{\eta}}\left(>0\right) \\
T &\equiv \frac{\rho\left(1-\alpha\right)}{\frac{n_1+n_2}{n_2}-\rho\left(1-\alpha\right)}\left(>0\right)
\end{aligned}$$

and c_1/g_1 is given by (5.26). Notice also that $\nu > 0$ whatever $\eta > 0$.

Assumption 6. *The density of immigrants is bounded from above:*

$$\frac{n_2}{n_1+n_2} < \nu$$

Assumption 6 is comfortably satisfied, in Western countries. For instance, in the logarithmic case ($\eta = 1$), the inequality simplifies to $n_2/\left(n_1+n_2\right) < \theta/\left(1+\theta\right)$ and it is verified if immigrants are less than half of the population (indeed, under the assumption of costs convexity, $\theta/\left(1+\theta\right) > 1/2$).

Assumption 6 also implies $Q > 0$. Therefore, $\varepsilon_{n_2^*\gamma} < 0$ and $\varepsilon_{n_2^*\mu} < 0$. As expected, an increase in the degree of intolerance towards immigrants (either γ or μ) has a negative impact on the optimal immigration policy: the stronger intolerance, the lower the number of welcomed immigrants.

From (5.53), it follows that $\varepsilon_{n_2^*\pi} > 0$ iff $\eta > 1$. In practice, an increase in fiscal participation has a positive impact on the optimal number of immigrants if and only if $\eta > 1$. Notice that $\eta > 1$ implies that the private and the public good are complements in the individual basket of consumption. Indeed, an increase in free riding $(1-\pi)$ has always a negative effect on the provision of the public good. When goods are complements, this entails a decrease in the marginal utility of private good consumption; and, thus, a decrease in the number of immigrant workers required for the production of both goods. To the contrary, if goods are substitutes, a decrease in the provision of public good rises the marginal utility of private good and incentives its production. The production of private good more than offsets the reduction of public good and the optimal number of (welcomed) immigrant workers increases.

Analogously, according to (5.53), the impact of τ on the optimal degree of openness (n_2^*) is positive if $\eta > 1$ and $\tau < (1-\alpha)(1-s)$. In this case, an increase in the tax rate, raises tax receipts and, thus, the provision of the public good. When goods are complements, this implies also an increase in the marginal utility of the private good and, eventually, an increase in the number of immigrant workforce employed in the production of both goods. Conversely, when $\tau > (1-\alpha)(1-s)$, a Laffer effect emerges: if the government increases the tax rate over this threshold level, its revenues fall. Less public good is produced and, because of complementarity, less private good. The desired amount of workforce shrinks.

The impact of congestion costs (ρ) on the optimal immigration policy is generally negative: the higher the number of immigrants, the lower natives' utility deriving from the public good. However, this result no longer holds and $\varepsilon_{n_2^*\rho}$ becomes positive if we assume unconventional degrees of substitutability, *i.e.*:

$$\eta < \frac{(1-\alpha)\ln(n_1+n_2)^\rho}{T+(1-\alpha)\ln(n_1+n_2)^\rho}$$

In this case, the lower the provision of public good, the higher the marginal utility of the private good and its production level: aggregate production increases and requires more immigrants.

Eventually, the impact of the preference for the private good (α) is ambiguous. For simplicity, we focus on the logarithmic case ($\eta = 1$) and we obtain $\varepsilon_{n_2^*\alpha} = QT\alpha/(1-\alpha) > 0$. Therefore, a stronger bias for the private good incentives its production and requires more immigrant workers to be hired.

5.4.3 Optimal policy mix

In the previous section, we have first computed the optimal fiscal policy for a given immigration policy and then, the optimal immigration policy for a given fiscal policy. We now focus on a joint optimal policy: we suppose that the policy maker sets both policies simultaneously. As a consequence, the optimization program becomes more sophisticated. The policy maker maximizes now the welfare function with respect to two variables at the same time: the number of welcomed immigrants and taxation.

In the case of homogeneous utility functions, the optimal joint policy (τ^*, n_2^*) satisfies the system of equations (5.36)-(5.47).

In the case of CES functions (technology and preferences), the optimal policy mix solves the system (5.38)-(5.49) or, equivalently:

$$\frac{\gamma\theta}{1-\mu\beta_1}\frac{n_1}{n_2^*(\tau)}\left[\frac{n_2^*(\tau)}{n_1+n_2^*(\tau)}\right]^{1+\theta}$$
$$= \frac{\left[\frac{\beta_1 n_1}{n_2^*(\tau)}\left(\frac{1-\varkappa}{\varkappa}\frac{s}{1-s}\right)^{\frac{\sigma}{1-\sigma}}\right]^{\frac{1-\eta}{\eta}}\left[1+\frac{1-s}{\sigma\tau}-\frac{s\sigma+(1-s)\pi}{s\sigma+(1-s)\pi\sigma}-\frac{\rho n_2^*(\tau)}{n_1+n_2^*(\tau)}\right]}{(1-\beta_1)^{\frac{1}{\eta}}\left(\frac{1}{\alpha}\left[\frac{1-s}{\sigma\tau}-\frac{s\sigma+(1-s)\pi}{s\sigma+(1-s)\pi\sigma}\right]\right)^{\frac{1-\eta}{\eta}\frac{\varsigma}{1-\varsigma}}\left[1+\frac{1-s}{\sigma\tau}-\frac{s\sigma+(1-s)\pi}{s\sigma+(1-s)\pi\sigma}\right]^{\frac{\eta-\varsigma}{\eta(1-\varsigma)}}} \tag{5.57}$$

where

$$n_2^*(\tau)$$
$$= \left[\frac{n_1\tau}{1-\tau}\frac{1-\beta_1(1-\delta)}{1-\beta_1}\frac{s+(1-s)\pi}{s}\left(\frac{1-\alpha}{\alpha}\left[\frac{1-s}{\sigma\tau}-\frac{s\sigma+(1-s)\pi}{s\sigma+(1-s)\pi\sigma}\right]\right)^{\frac{\varsigma}{\varsigma-1}}\right]^{\frac{1}{\rho}}-n_1 \tag{5.58}$$

and $s=\hat{s}(\tau)$ is explicitly given by (5.45) (see the Appendix).

Notice that now (5.57) is a single equation in the unknown τ, where there are only exogenous parameters. It is straightforward to compute a numerical solution τ^*. The optimal policy mix is eventually given by $(\tau^*, n_2^*(\tau))$.

Concerning the current consumption of composite good φ, the (intertemporal) substitution and income effects cancel each other out in the case $\eta = 1$ (logarithmic case) and (5.57) simplifies:

$$\frac{\gamma\theta}{1-\mu\beta_1}\frac{n_1}{n_2^*(\tau)}\left[\frac{n_2^*(\tau)}{n_1+n_2^*(\tau)}\right]^{1+\theta} = (1-\beta_1)\frac{1+\frac{1-s}{\sigma\tau}-\frac{s\sigma+(1-s)\pi}{s\sigma+(1-s)\pi\sigma}-\frac{\rho n_2^*(\tau)}{n_1+n_2^*(\tau)}}{1+\frac{1-s}{\sigma\tau}-\frac{s\sigma+(1-s)\pi}{s\sigma+(1-s)\pi\sigma}} \tag{5.59}$$

In the logarithmic case, A and $\varkappa$ do not appear in either (5.58) or (5.59). Therefore, we can fix s by using empirical estimates and calibrate A or $\varkappa$, according to (5.45).

In the case of a CES technology, when $\sigma > 1$ firms can produce with either capital or labor only (see (5.5)). This is why both inputs are unessential; thus, when we solve (5.57), we can obtain corner solutions.

In order to provide some quantitative insights we now focus on Cobb-Douglas functions for technology and preferences. Notice that under a Cobb-Douglas technology, both inputs are necessary (Inada conditions hold) and interior solutions are more likely. Moreover, as shown above, in the Cobb-Douglas case (5.4)-(5.39) s is an exogenous parameter and τ^* no longer depends on n_2: τ^* is explicitly given by (5.40).

By replacing first (5.40) in (5.26) and then (5.22) and (5.26) in (5.50), we can rewrite the immigration arbitrage as a single equation in the unknown n_2^*.[13] As a benchmark case, we refer to yearly data and we assume a relatively small bias towards private goods, an intermediate degree of "pureness" of the public good and a relative strong degree of rule of low in the host country (implying, in turn, a low degree of free riding). We also calibrate our model so as to have significant cultural diversity costs.[14]

Figure 5.1 represents natives' welfare as a function of both taxation and immigrants, for a

[13] n_2^* must satisfy also the second order conditions (see the Appendix).

[14] Let us set $\beta_1 = 0.95$ and $\delta = 0.10$ and normalize A to 1. Moreover, we assume $\eta = 1.1$ (slight intertemporal substitution corresponding to a weak complementarity between private and public good), $\alpha = 0.6$ (a relatively small bias towards private goods), $\mu = 0.9$ (a slow decline in hostility), $\pi = 0.7$ (a relatively good rule of law in the host country), $\rho = 0.5$ (an intermediate degree of rivality of public good), $\theta = 1.5$ (weak concavity of costs), $\gamma = 50$ (significant cultural costs). For simplicity, we fix the total population size to $n_1 + n_2 = 100$.

benchmark set of parameters consistent with the above assumptions.

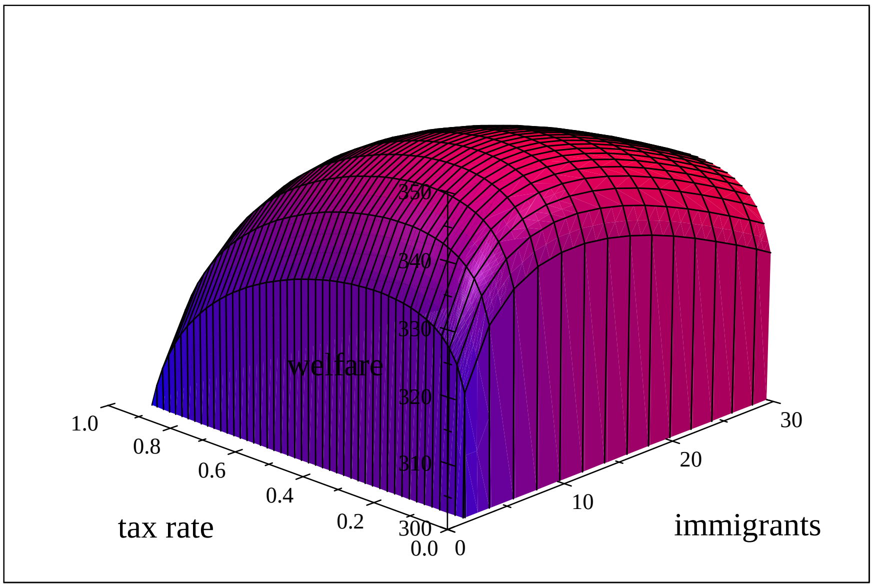


Figure 5.1. Welfare

Our baseline calibration suggests that the optimal immigration policy consists in letting about 10% of total population being foreign born; the optimal fiscal taxation (in presence of a relatively low preference towards the private good) is around 40%.

5.5 Local dynamics

The stability properties of the steady state and the occurrence of local bifurcations can be evaluated by linearizing the dynamic system (5.17)-(5.18) around the steady state and computing the Jacobian matrix J.

Proposition 16 *Local dynamics are represented by the linear system* $(dk_{t+1}/k, dc_{1t+1}/c_1)^T = J\,(dk_t/k, dc_{1t}/c_1)^T$. *$J$ is the Jacobian matrix evaluated at the steady state:*

$$J = \frac{1}{\beta_1}\begin{bmatrix} 1+\xi\varepsilon_r & \beta_1 - 1 \\ E\beta_1 - (1+\xi\varepsilon_r)\left(E + \frac{\xi\varepsilon_r}{\varepsilon_{cc}}\right) & \beta_1 + (1-\beta_1)\left(E + \frac{\xi\varepsilon_r}{\varepsilon_{cc}}\right) \end{bmatrix} \tag{5.60}$$

where $\xi \equiv 1 - \beta_1 (1 - \delta)$ *and*

$$E \equiv \frac{\varepsilon_{cg}}{\varepsilon_{cc}} \frac{s (1 + (1 - \pi) \varepsilon_r)}{s + (1 - s) \pi} \tag{5.61}$$

Proof. See the Appendix.

In the above Proposition, $-1/\varepsilon_{cc} \equiv -u_{1c}/(u_{1cc}c_1) > 0$ denotes the citizens' elasticity of intertemporal substitution in private consumption and $\varepsilon_{cg} \equiv g_1 u_{1cg}/u_{1c}$ is a cross elasticity which captures the interplay between the private and the public good within preferences. When $\varepsilon_{cg} > 0$, goods are rather complementary, while, when $\varepsilon_{cg} < 0$, they are rather substitutable.

As emphasized by Grandmont, Pintus and de Vilder (1998), the stability properties of the system (the location of the eigenvalues with respect to the unit circle) can be better characterized in the (T, D)-plane. In the following, we exploit the fact that the trace T and the determinant D of J are the sum and the product of the eigenvalues, respectively.

More explicitly, we evaluate the characteristic polynomial $P(x) \equiv x^2 - Tx + D$ at $x = -1$ and $x = 1$ and we plot the corresponding lines $P(-1) = 0$, $P(1) = 0$ in the (T, D)-plane, that is $D = -T - 1$ and $D = T - 1$ with intersection A. Furthermore we consider the line $D = 1$ and we call B and C its intersections with the lines $D = -T - 1$ and $D = T - 1$, respectively. Along the line AC, one eigenvalue is equal to 1. Along the line AB, one eigenvalue is equal to -1. On the segment (BC) without the endpoints, the two eigenvalues are complex and conjugate with unit modulus, *i.e.* $D = 1$ and $|T| < 2$. Therefore, inside the triangle ABC, the steady state is a sink, *i.e.* locally indeterminate ($D < 1$ and $|T| < 1 + D$). It is a saddle point if (T, D) lies on the right sides of both AB and AC or on the left sides of both of them ($|1 + D| < |T|$). It is a source otherwise (see Figure 5.2).

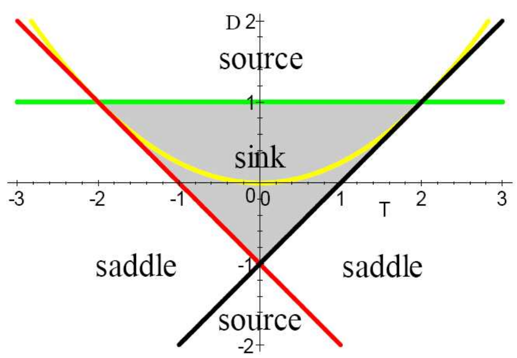


Figure 5.2. Trace and determinant

A (local) bifurcation arises when an eigenvalue crosses the unit circle, that is, when the pair (T, D) crosses one of the loci AB, AC or $[BC]$. (T, D) depends on the structural parameters. We choose a parameter of interest and we study how (T, D) moves with it in the (T, D)-plane. More precisely, according to the changes in the bifurcation parameter, a saddle node bifurcation (generically) occurs when (T, D) goes through AC, a flip bifurcation (generically) arises when (T, D) crosses AB, whereas a Hopf bifurcation (generically) emerges when (T, D) goes through the segment $[BC]$, endpoints included.

Becker and Foias (1994) choose the elasticity of capital-labor substitution as bifurcation parameter: σ plays a crucial role in explaining the emergence of endogenous cycles. In line with Becker and Foias, we determine the critical value of σ.

The occurrence of persistent fluctuations depends on the peculiar characteristics of the market. The following Assumption allows us to link market fundamentals to the emergence of cycles.

Assumption 7 *(substitutability or weak complementarity)* $\varepsilon_{cg} < \varepsilon_{cg}^*$, *where*

$$\varepsilon_{cg}^* \equiv -\varepsilon_{cc} \frac{s + (1-s)\pi}{s} \frac{1+\beta_1}{1-\beta_1} (> -\varepsilon_{cc} > 0)$$

The following Proposition characterizes the bifurcation value for σ.

Proposition 17 *Under Assumption 7, the steady state is a source if and only if* $\sigma < \sigma^*$, *where*

$$\sigma^* \equiv (1-s) \frac{\frac{\xi}{2} + \frac{\xi}{1-\beta_1}\varepsilon_{cc} + \frac{(1-\pi)s}{\pi+(1-\pi)s}\varepsilon_{cg}}{\frac{1+\beta_1}{1-\beta_1}\varepsilon_{cc} + \frac{s}{\pi+(1-\pi)s}\varepsilon_{cg}} \tag{5.62}$$

Otherwise it is a saddle. When σ *crosses* σ^*, *the economic system generically undergoes a flip bifurcation, that is, cycles of period two emerge.*

Proof. See the Appendix.

We notice that, under Assumption 7, σ^* is strictly positive if and only if its numerator is negative, that is, if and only if

$$\varepsilon_{cg} < \varepsilon_{cg}^{**} \equiv -\frac{\pi + (1-\pi)s}{(1-\pi)s} \left(\frac{\xi}{2} + \frac{\xi}{1-\beta_1}\varepsilon_{cc} \right) \tag{5.63}$$

The right-hand side in (5.63) is positive if the citizens' intertemporal substitution in consumption is sufficiently low: $0 < -1/\varepsilon_{cc} < 2/(1-\beta_1)$. However, this range is quite broad and includes the set of empirical values. In this case, the more honest immigrants (*i.e.*, the closer the fiscal participation π to one), the larger the range for ε_{cg}.

The bifurcation value σ^* is a function of the degree of immigrants' free riding; this suggests that the extent of the black labor market could play a destabilizing effect in our economy. The following Lemma characterizes the linkage between free riding and the emergence of cycles.

Lemma 18 *Under Assumption 7, the degree of free riding (or the extent of black labor market)* $1-\pi$ *promotes the occurrence of cycles if and only if*

$$(1-\sigma^*)\,\varepsilon_{cg} > 0 \tag{5.64}$$

Proof. See the Appendix.

When condition (5.64) is satisfied, the black labor market or the tax evasion must be kept under control; indeed, they translate into potential sources of inefficient fluctuations.

We now focus on the Cobb-Douglas utility function (5.39) so as to isolate the effect of black labor markets; in this case it is possible to find explicit results and explain why cycles arise.

Proposition 19 *In the Cobb-Douglas case (5.39), the extent of the black labor market has a destabilizing effect, that is* $1-\pi$ *promotes the occurrence of cycles, if and only if the complementarity in consumption prevails* ($\eta > 1$)*.*

Proof. See the Appendix.

We observe that in this case neither the fiscal nor the immigration policy have any effect on the occurrence of cycles; under the assumption of the model, there is no room for stabilization interventions. Having said that, the extent of the black-labor market plays a significant role and there is scope for policy intervention (beyond the assumptions of the model).

The occurrence of cycles can be explained as follows. In the Cobb-Douglas case, when $\eta > 1$, public and private goods are complements (that is, the consumption of public good increases the marginal utility of private good and *viceversa*). But $\eta > 1$ also means that the intertemporal substitution dominates the income effect. If k_t increases, capital income also increases at time t, entailing both an increase in taxes and public good provision. Because of the complementarity between c_1 and g_1, we observe also an increase in private consumption.[15] This eventually reduces investment, and thus k_{t+1} (according to the budget constraint). This negative impact of k_t on k_{t+1} is amplified by the black labor market (free riding). The reason is that, if free riding is significant, the provision of public good mostly weights on natives' capital income: the higher the contribution of capital income to the provision of the public good, the stronger the effect.

5.6 Sorting out shocks in a Cobb-Douglas economy

The saddle-path stability is a central feature of the Ramsey model and entails the uniqueness of equilibrium under rational expectations.[16]

Along the linearized saddle path, capital and consumption evolve according to the following system:

$$k_t = k + (k_0 - k)\,\lambda_1^t \tag{5.65}$$

$$c_{1t} = c_1 + \left[\frac{\beta_1}{1-\beta_1}(\lambda_2 - 1)\,\lambda_1 - E\right](k_0 - k)\,\lambda_1^t \tag{5.66}$$

where k_0 is the initial level of aggregate capital, while k and c_1 are the steady state of aggregate capital intensity and consumption, respectively. E is given by (5.61), λ_1 and λ_2 are respectively the stable and the unstable eigenvalue of the Jacobian matrix evaluated at the steady state (see the Appendix).

If policy parameters are subject to a shock, the steady state of the system shifts to a new long-run equilibrium. The steady-state level of capital before the shock becomes the initial

[15] The elasticity of intertemporal substitution η of the composite good φ does not coincide with the partial elasticities of intertemporal substitution in private and public good $-1/\varepsilon_{cc}$ and $-1/\varepsilon_{gg}$: in the Cobb-Douglas case (5.39):

$$\varepsilon_{cc} \equiv \frac{cu_{cc}}{u_c} = -\left[1 - \alpha\left(1 - \frac{1}{\eta}\right)\right] < 0$$

$$\varepsilon_{cg} \equiv \frac{gu_{cg}}{u_c} = (1-\alpha)\left(1 - \frac{1}{\eta}\right) > 0 \text{ iff } \eta > 1$$

However, when $\alpha \to 1$, the composite good turns out to be the private good and we recover the equality $\eta = -1/\varepsilon_{cc}$.

[16] See, among others, Blanchard and Quah (1989).

condition of the transition to the new steady state. Starting from the intersection between the vertical line in the (k_t, c_{1t})-plane, policy shocks trigger jumps in consumption; the system moves thus along a new saddle path, while capital gradually accumulate/decrease (see the Appendix, for details).

In order to have some quantitative insights on the dynamics associated to the adjustment following a policy change, we now focus on Cobb-Douglas production and utility functions (see expressions (5.4) and (5.39)).

As above, we set $A = 1$, $s = 0.3$, $\beta = 0.95$ and $\delta = 0.10$ (yearly data).[17] We assume a degree of complementarity between the private and public good ($\eta = 1.1$) and a slight preference for the private good ($\alpha = 0.6$). The immigrants/natives ratio is calibrated from OECD data on foreign population as a percentage of total population. We consider the average during the 1990s of US foreign population as a percentage of total population; we thus let $n_2/n_1 = 10/90$. The income tax is then calculated as an average of total tax income in OECD countries in the year 2000 ($\tau = 0.38$).

Figure 5.3 shows the effects on capital and consumption of a decrease in the income tax level from $\tau = 0.381$ to $\tau' = 0.380$. As expected, the shock has a positive effect on long-run values of both consumption and aggregate capital.

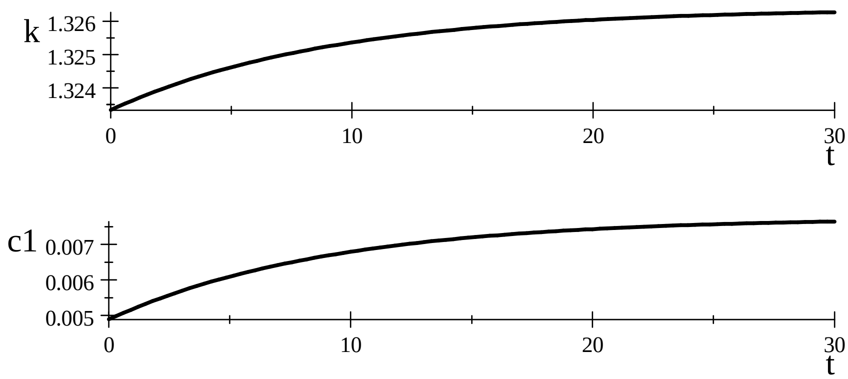


Figure 5.3. Shock on the fiscal policy

We may question whether these dynamics are affected by a change in the number of immigrants. We observe that a policy mix requiring a joint decrease in the taxation rate (as

[17]See, for instance, Benhabib and Farmer (1996).

above, from $\tau = 0.381$ to $\tau' = 0.38$) and in the number of immigrants (from $n_2/n_1 = 10/90$ to $n_2'/n_1 = 8/92$) does not affect capital intensity, but shifts the consumption path downwards, as illustrated in Figure 5.4: the smaller the amount of immigrants, the smaller the initial and final consumption levels.

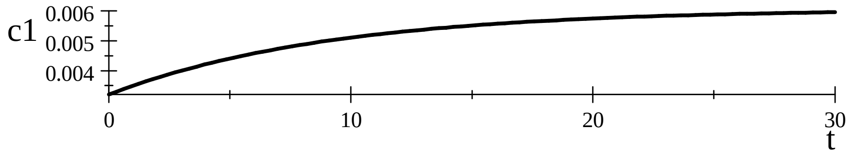


Figure 5.4. Shocks on the fiscal and immigration policy

5.7 Concluding remarks

In this work, we have characterized optimal fiscal and immigration policies in a dynamic framework of general equilibrium. We have accounted for the possibility of free riding in immigrants' contribution to government revenues and for natives' intolerance towards immigrants. We have assumed the perspective of a government who provides a rival public good and maximizes the welfare of natives only.

Our work proves that, in absence of costs of cultural heterogeneity, it is optimal to keep frontiers open; on the other hand, in presence of costs of cultural heterogeneity, the optimal immigration policy is an interior solution. In order to demonstrate it, we have introduced isoelastic functional forms and shown that an increase in the degree of intolerance towards immigrants has a negative impact on the optimal number of welcomed immigrants. We also have shown that, under Cobb-Douglas specifications, the effect of both an increase in taxation and in the degree of free riding on optimal quotas is not univocal and depends on the substitutability between the private good and the public good in natives' consumption basket.

An increase in free riding entails less public good provision; if goods are complement, a decrease in the public good implies a decrease in the marginal utility of private consumption. Therefore, firms produce less private good and less immigrants workers are needed. To the contrary, if goods are substitute, a decrease in the provision of the public good rises the marginal utility of private the good. This prompts to produce more private good, and thus, to hire (*i.e.*, welcome) a larger number of immigrants.

Analogously, if goods are complements and the tax rate sufficiently low, an increase in taxation (*i.e.*, in the provision of public good) has a positive effect on the marginal utility of private good and the optimal number of immigrants; to the contrary, if the tax rate is high enough, a further increase in taxation can dampen government revenues (Laffer effect), the provision of public good, the consumption of private good and, eventually, the number of foreign workers.

Surprisingly, the characterization of the optimal fiscal policy shows that the degree of intolerance does not affect optimal taxation. If the ratio between the elasticities of utility with respect to public and private consumption is constant, also the degree of impureness of the public good has no impact on the optimal taxation rate. Indeed, while the degree of congestion dampens the utility deriving from consumption, the provision of the public good depends on agents' incomes. Moreover, under Cobb-Douglas technology and preferences, the number of immigrants does not affect optimal taxation while the impact of the degree of free riding depends on factor substitutability.

In this work, we prove the existence of persistent fluctuations for plausible parameter values and we show that the extent of black labor market can promote cycles, when private and public goods are complements.

Finally, our analysis studies the local dynamics following shocks in the policy parameters and shows that changes in the immigration policy do not affect long-run capital intensity. However, if the number of immigrants increases, individual capital and private consumption increase as well. To the contrary, a negative shock in the tax rate implies a lower level of both capital intensity and individual consumption in the long run.

5.8 Appendix

Optimal immigration policy

In order to compute analytically the government optimization program, we maximize the welfare function evaluated at the steady state (see (5.29)), with respect to the policy n_2:

$$\frac{\partial \omega}{\partial n_2} = \frac{1}{n_2}\frac{u_1}{1-\beta_1}\left[\varepsilon_{u_1 c_1} + \frac{n_1 + (1-\rho)\, n_2}{n_1 + n_2}\varepsilon_{u_1 g_1}\right] - \frac{n_1}{(n_1+n_2)^2}\frac{v_1'}{1-\mu\beta_1} = 0 \tag{5.67}$$

In order to solve (5.67), we consider explicit utility functions. In the case of a homogeneous utility function (5.33), substituting (5.34) and (5.35) in (5.67), we obtain the optimal arbitrage (5.47).

In the CES case (technology and preferences), (5.47) becomes

$$\frac{u_1}{1-\beta_1}\frac{\eta-1}{\eta}\left(1 - \frac{\rho n_2}{n_1+n_2}\left[1 + \frac{\alpha}{1-\alpha}\left(\frac{c_1}{g_1}\right)^{1-1/\varsigma}\right]^{-1}\right) = \frac{n_1}{n_1+n_2}\frac{v_1}{1-\mu\beta_1}\varepsilon_{v_1} \tag{5.68}$$

By replacing (5.33) and (5.37) in (5.68) and by using (5.48), we obtain (5.49).

For simplicity, we focus on the case of Cobb-Douglas technology and preferences; we obtain the immigration arbitrage (5.50) by setting first $\varsigma = 1$ in (5.68) and substituting φ and v_1 with (5.39) and (5.48), respectively, and g_1 with (5.25).[18] We compute the impact of the fiscal pressure τ and the fundamental parameters α, μ, γ, π, ρ on the optimal immigration policy n_2^*.

Equation (5.50) can be solved numerically with respect to n_2. However, since we are interested in analytical results, we apply the Implicit Function Theorem to (5.50). Indeed, we do not need an explicit solution n_2^* to study how changes in the fiscal pressure τ and in the fundamental parameters α, μ, γ, π, ρ affect the optimal immigration policy n_2^*.

By totally differentiating (5.50) with respect to $(n_2, \tau, \alpha, \mu, \gamma, \pi, \rho)$ around n_2^* and reducing differentials into elasticities, we obtain the solutions (5.51) to (5.56).

Optimal policy mix

In the case of CES functions, the optimal policy mix solves the system (5.38)-(5.49). From

[18] Notice also that under (5.4), (5.39), (5.48), the second-order conditions for welfare maximization are satisfied, since, according to (5.67), $\partial\omega/\partial n_2$ is decreasing. Equivalently, the left and the right-hand sides in (5.47) are respectively decreasing and increasing.

(5.28) and (5.38), we obtain

$$\left(\frac{c_1}{g_1}\right)^* = \left(\frac{1-\alpha}{\alpha}\left[\frac{1-s}{\sigma\tau} - \frac{s\sigma + (1-s)\,\pi}{s\sigma + (1-s)\,\pi\sigma}\right]\right)^{\frac{\varsigma}{\varsigma-1}} \tag{5.69}$$

where $s = \hat{s}\,(\tau)$ is explicitly given by (5.45). By using (5.26) and (5.69), we compute n_2^* as a function of τ (see (5.58).

Then, by substituting (5.69) and $n_2^*\,(\tau)$ in (5.49), and noticing that, from (5.6),

$$k = \left(\frac{1-s}{s}\frac{\varkappa}{1-\varkappa}\right)^{\frac{\sigma}{1-\sigma}}$$

we obtain (5.57).

Proof of Proposition 16

Notice that $w/\,(rk) = 1/s - 1$, $\varepsilon_w = -\varepsilon_r s/\,(1-s)$ and

$$\frac{\tau r k}{g_1} = \frac{s}{s + (1-s)\,\pi}\frac{(n_1+n_2)^{\rho}}{n_2}$$

we can thus write the linear system as follows:

$$\left(E + \frac{\xi\varepsilon_r}{\varepsilon_{cc}}\right)\frac{dk_{t+1}}{k} + \frac{dc_{1t+1}}{c_1} = E\frac{dk_t}{k} + \frac{dc_{1t}}{c_1} \tag{5.70}$$

$$\beta_1\frac{dk_{t+1}}{k} = (1+\xi\varepsilon_r)\,\frac{dk_t}{k} - (1-\beta_1)\,\frac{dc_{1t}}{c_1} \tag{5.71}$$

■

Proof of Proposition 17

The determinant D and the trace T of the Jacobian matrix J are the following:

$$\begin{aligned} D &= \frac{1+\xi\varepsilon_r}{\beta_1} + \frac{1-\beta_1}{\beta_1}E \\ T &= 1 + D + \frac{1-\beta_1}{\beta_1}\frac{\xi\varepsilon_r}{\varepsilon_{cc}} \end{aligned}$$

First, we observe that

$$D = T - 1 - \frac{1-\beta_1}{\beta_1}\frac{\xi\varepsilon_r}{\varepsilon_{cc}} < T - 1$$

because $\varepsilon_r, \varepsilon_{cc} < 0$.

In other terms, the economy never lies inside the sink-stability triangle ABC. In order to know whether the economy is characterized by saddle-path stability or a source configuration, we need to locate (T, D) with respect to the line AB, that is to check the inequality $D < -T - 1$ (source case). This inequality can be equivalently written as follows:

$$E < -\frac{1}{2}\frac{\xi\varepsilon_r}{\varepsilon_{cc}} - \frac{1+\beta_1+\xi\varepsilon_r}{1-\beta_1}$$

or, more explicitly, taking into account definition (5.61),

$$\frac{\varepsilon_{cg}}{\varepsilon_{cc}}\frac{s\left(1+(1-\pi)\,\varepsilon_r\right)}{s+(1-s)\,\pi} < -\frac{1}{2}\frac{\xi\varepsilon_r}{\varepsilon_{cc}} - \frac{1+\beta_1+\xi\varepsilon_r}{1-\beta_1} \tag{5.72}$$

If (5.72) holds with equality, we generically have a flip bifurcation. More precisely, by solving (5.72) with respect to a parameter of interest we find the critical value for this parameter (through which the economic system undergoes a flip bifurcation).

When we consider (5.3), inequality (5.72) can be rewritten as:

$$\left(\frac{\xi}{2} + \frac{\xi}{1-\beta_1}\varepsilon_{cc} + \frac{(1-\pi)\,s}{\pi+(1-\pi)\,s}\varepsilon_{cg}\right)\frac{1-s}{\sigma} < \frac{1+\beta_1}{1-\beta_1}\varepsilon_{cc} + \frac{s}{\pi+(1-\pi)\,s}\varepsilon_{cg}$$

The right-hand side is positive if and only if $\varepsilon_{cg} \geq \varepsilon_{cg}^*$, that is a very demanding restriction in terms of cross effects.

To the contrary, a plausible condition is given by Assumption 7. Under Assumption 7, (5.72) becomes equivalent to (5.62). ■

Proof of Lemma 18

In order to analyze this effect, we compute the derivative of σ^* with respect to π. If free riding $(1-\pi)$ increases σ^*, the economy is more likely to experience two-period cycles around a source. From (5.62), we obtain:

$$\frac{\partial\sigma^*}{\partial\pi} = (\sigma^*-1)\,\frac{1-s}{\pi+(1-\pi)\,s}\,\frac{\frac{s}{\pi+(1-\pi)s}\varepsilon_{cg}}{\frac{1+\beta_1}{1-\beta_1}\varepsilon_{cc} + \frac{s}{\pi+(1-\pi)s}\varepsilon_{cg}}$$

Under Assumption 7, free riding $1-\pi$ can destabilize the economy by widening the range of deterministic cycles around a source $(0, \sigma^*)$; it makes cycles more likely, if and only if (5.64) holds.

More explicitly, condition $\sigma^* < 1$ is equivalent to

$$\frac{1+\beta_1-(1-s)\,\xi}{1-\beta_1}\varepsilon_{cc}+\frac{s\left[1-(1-\pi)(1-s)\right]}{\pi+(1-\pi)\,s}\varepsilon_{cg}<(1-s)\,\frac{\xi}{2}$$

■

Proof of Proposition 19

The critical bifurcation value (5.62) becomes:

$$\sigma^* = (1-s)\,\frac{\frac{\xi}{2}+\frac{(1-\pi)s}{\pi+(1-\pi)s}(1-\alpha)\left(1-\frac{1}{\eta}\right)-\frac{\xi}{1-\beta_1}\left[1-\alpha\left(1-\frac{1}{\eta}\right)\right]}{\frac{s}{\pi+(1-\pi)s}(1-\alpha)\left(1-\frac{1}{\eta}\right)-\frac{1+\beta_1}{1-\beta_1}\left[1-\alpha\left(1-\frac{1}{\eta}\right)\right]} \tag{5.73}$$

When the utility function is a Cobb-Douglas, Assumption 7 translates to:

$$\frac{1}{\eta} > \frac{1-\frac{1+\beta_1}{1-\beta_1}\frac{\pi+(1-\pi)s}{s}}{1+\frac{\alpha}{1-\alpha}\frac{1+\beta_1}{1-\beta_1}\frac{\pi+(1-\pi)s}{s}} \tag{5.74}$$

which is always satisfied (the right-hand side is negative). By substituting (5.73) in (5.64), one proves that, under inequality (5.74), condition (5.64) is equivalent to $\eta > 1$. From Lemma 18, follows the corollary. ■

Computing the trajectory toward the steady state

In our two-dimensional dynamics, the saddle path in the (c_{1t}, k_t)-plane is associated to the stable eigenvalue (say λ_1), while the unstable manifold corresponds to the unstable eigenvalue (say λ_2). We have proved that, when a parameter crosses its critical value, λ_1 crosses -1 and the system generically undergoes a flip bifurcation.

When we focus on the saddle-path stability, we are first interested in computing the trajectory along the saddle path to the steady state.

Let

$$J = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \tag{5.75}$$

be the Jacobian matrix (5.60).

In our case, the eigenvalues are real and distinct, because $D < T - 1$ (if saddle, obviously; if source, since $\lambda_1 \lambda_2 = D < -1 < T^2/4$).

We then decompose J as follows:

$$J = \begin{bmatrix} \lambda_1 - d & \lambda_2 - d \\ c & c \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} \lambda_1 - d & \lambda_2 - d \\ c & c \end{bmatrix}^{-1} \equiv V \Lambda V^{-1}$$

where $\lambda_1 = \left(T - \sqrt{T^2 - 4D}\right)/2$ and $\lambda_2 = \left(T + \sqrt{T^2 - 4D}\right)/2$ are the eigenvalues (with $T = a + d$ and $D = ad - bc$).

The saddle path is nonlinear, but we can approximate it with a linearized saddle path tangent to the nonlinear one in the steady state.

The (linearized) saddle path is characterized by the fact that if an economy is in a point on this path, it will converge to the steady state.

Formally, $\lim_{t \to +\infty} x_t = x$, where $x_t \equiv (k_t, c_{1t})^T$, or, approximately,

$$\lim_{t \to +\infty} (x_t - x) \approx \lim_{t \to +\infty} J^t (x_0 - x) = \lim_{t \to +\infty} V \Lambda^t V^{-1} (x_0 - x) = 0$$

where x_0 and x are, respectively, the initial condition and the steady state.

We observe that $J^t (x_0 - x) \to 0$ if and only if $\Lambda^t V^{-1} (x_0 - x) \to 0$. In the saddle case, $|\lambda_1| < 1$ and $|\lambda_2| > 1$. Thus, convergence requires the second component of the vector $V^{-1} (x_0 - x)$ to be zero. More explicitly, we require

$$\begin{aligned} V^{-1} (x_0 - x) &= \frac{1}{c (\lambda_1 - \lambda_2)} \begin{bmatrix} c & d - \lambda_2 \\ -c & \lambda_1 - d \end{bmatrix} \begin{bmatrix} k_0 - k \\ c_{10} - c_1 \end{bmatrix} \\ &= \begin{bmatrix} \frac{k_0 - k}{\lambda_1 - \lambda_2} + \frac{d - \lambda_2}{c} \frac{c_{10} - c_1}{\lambda_1 - \lambda_2} \\ 0 \end{bmatrix} \end{aligned}$$

that is, $(\lambda_1 - d)(c_{10} - c_1) = c (k_0 - k)$.

Therefore, in order to neutralize the action of the unstable eigenvalue, the starting point

(k_0, c_{10}) must lie on the saddle path, so as to satisfy the following equation:

$$c_{10} = c_1 + \frac{c}{\lambda_1 - d}(k_0 - k) \tag{5.76}$$

We notice that k_t is a predetermined variable (state), while c_{1t} is a jump variable (control). A rational expectation equilibrium requires c_{10} to adjust in order to ensure that (k_0, c_{10}) belongs to the saddle path. In other terms, c_{10} adjusts to satisfy (5.76), given the initial condition k_0.

The (approximated) trajectory is now computed, given the hypothesis of rational expectations and the initial condition k_0: $x_t \approx x + J^t(x_0 - x)$, that is:

$$\begin{bmatrix} k_t \\ c_{1t} \end{bmatrix} \approx \begin{bmatrix} k \\ c_1 \end{bmatrix} + \begin{bmatrix} \lambda_1 - d & \lambda_2 - d \\ c & c \end{bmatrix} \begin{bmatrix} \lambda_1^t & 0 \\ 0 & \lambda_2^t \end{bmatrix} \begin{bmatrix} \frac{k_0 - k}{\lambda_1 - \lambda_2} + \frac{d - \lambda_2}{c}\frac{c_{10} - c_1}{\lambda_1 - \lambda_2} \\ 0 \end{bmatrix}$$

Substituting $c_{10} - c_1 = c(k_0 - k)/(\lambda_1 - d)$ (equation (5.76)) and solving, we get the trajectory along the saddle path:

$$\begin{aligned} k_t &= k + (k_0 - k)\lambda_1^t & (5.77) \\ c_{1t} &= c_1 + \frac{c}{\lambda_1 - d}(k_0 - k)\lambda_1^t & (5.78) \end{aligned}$$

According to (5.60), we also obtain:

$$\begin{aligned} c &\equiv E - \frac{1 + \xi\varepsilon_r}{\beta_1}\left(E + \frac{\xi\varepsilon_r}{\varepsilon_{cc}}\right) \\ d &\equiv 1 + \frac{1 - \beta_1}{\beta_1}\left(E + \frac{\xi\varepsilon_r}{\varepsilon_{cc}}\right) \end{aligned}$$

Tedious computations give

$$\frac{c}{\lambda_1 - d} = \frac{\beta_1}{1 - \beta_1}(D - \lambda_1) - E \tag{5.79}$$

or, equivalently,

$$\frac{c}{\lambda_1 - d} = \frac{\beta_1}{1 - \beta_1}(\lambda_2 - 1)\lambda_1 - E$$

Therefore, the trajectory along the saddle path is explicitly described by (5.65-5.66).

Sorting out shocks

Consider first a change in the immigration policy such that the number of immigrants varies from n_2 to n_2'. As mentioned before, we observe that the stationary level of aggregate capital does not depend on n_2 (see (5.19)), while c_1 depends on n_2 according to (5.24). However, a shock in the number of immigrants does not generate a transition dynamics: k_t is a state variable and c_{1t} is a jump variable, which adjusts instantaneously so as to ensure the saddle-path stability. More precisely, ex-ante and ex-post steady states lie on the same vertical line in the (k_t, c_{1t})-plane. The saddle path translates vertically from $c_{10} = c_1 + c(k_0 - k) / (\lambda_1 - d)$ to $c_{10} = c_1' + c(k_0 - k) / (\lambda_1 - d)$.[19]

The native's consumption (given by (5.24)), allocation of capital and public good jump from c_1, k_1, g_1 to

$$\begin{aligned} c_1' &= \frac{1-\beta_1}{\beta_1}\frac{n_2'}{n_1}k \\ k_1' &= \frac{n_2'}{n_1}k \\ g_1' &= \tau(rk + \pi w)\frac{n_2'}{(n_1+n_2')^{\rho}} \end{aligned}$$

respectively.

Consider now a shock in the fiscal policy consisting in an increase (decrease) in the taxation rate from τ to τ'. Such policy change implies in the long run a smaller (larger) amount of aggregate capital and individual consumption, k' and c_1' respectively. New steady-state values are:

$$\begin{aligned} k' &= r^{-1}\left(\frac{1/\beta_1 - 1 + \delta}{1-\tau'}\right) \\ c_1' &= \frac{1-\beta_1}{\beta_1}\frac{n_2}{n_1}k' \end{aligned}$$

while the old stationary capital becomes now an initial condition:

$$k_0 = k = r^{-1}\left(\frac{1/\beta_1 - 1 + \delta}{1-\tau}\right)$$

[19] When the utility function is isoelastic, the Jacobian matrix (and thus, c, d and λ_1) no longer depends on n_2 and, thus, the slope of the saddle-path does not change.

According to (5.77)-(5.78) and (5.79), the trajectory along the new saddle path becomes

$$\begin{aligned} k_t &= k' + \left(k - k'\right) \lambda_1'^t \\ c_{1t} &= c_1' + \left[\frac{\beta_1}{1-\beta_1}\left(D' - \lambda_1'\right) - E'\right]\left(k - k'\right) \lambda_1'^t \end{aligned}$$

where now D', λ_1' and E' are evaluated in the new steady state.

Focus now on a joint shock on fiscal and immigration policy: $(\tau, n_2) \rightarrow (\tau', n_2')$. We observe that a shock on the number of immigrants has now an impact on the transition generated by a shock on the tax pressure. A right-wing party could for instance decide to lower the tax rate and the number of immigrants at the same time. The transition path **is:**

$$\begin{aligned} k_t &= k' + \left(k - k'\right) \lambda_1'^t \\ c_{1t} &= c_1' + \left[\frac{\beta_1}{1-\beta_1}\left(D' - \lambda_1'\right) - E'\right]\left(k - k'\right) \lambda_1'^t \end{aligned}$$

where, in presence of an isoelastic utility function, D', λ_1' and E' depend only on k', while

$$\begin{aligned} k' &= r^{-1}\left(\frac{1/\beta_1 - 1 + \delta}{1 - \tau'}\right) \\ c_1' &= \frac{1-\beta_1}{\beta_1}\frac{n_2'}{n_1}k' \end{aligned}$$

Notice however that the speed of convergence is only affected by the fiscal policy, because it only depends on λ_1. More precisely, the non-linear dynamics are approximated in a neighborhood of the steady state by the saddle path:

$$k_t - k' = \left(k - k'\right) \lambda_1^t \tag{5.80}$$

where the initial condition k is assumed to belong to a neighborhood of k'.[20]

Joint conditions for welfare maximization

[20] In order to calculate the time interval to cover a share $\varkappa \in (0,1)$ of the distance between the initial condition and the steady state, we must solve the equation: $\varkappa = (k_t - k)/(k' - k) = 1 - \lambda_1^t$, derived from (5.80). We obtain $t = \ln(1-\varkappa)/\ln\lambda_1$, that is an inverse measure of the speed of convergence. For instance, the system needs $t = -\ln 2/\ln\lambda_1$ to cover half of the distance from the old steady state to the new one.

The first order conditions for maximization become $(\partial\omega/\partial\tau, \partial\omega/\partial n_2) = (0,0)$, that is

$$\begin{aligned}
\frac{\partial u_1}{\partial c_1} c_1 \varepsilon_{c_1\tau} + \frac{\partial u_1}{\partial g_1} g_1 \varepsilon_{g_1\tau} &= 0 && (5.81)\\
\frac{\partial u_1}{\partial c_1} c_1 \varepsilon_{c_1 n_2} + \frac{\partial u_1}{\partial g_1} g_1 \varepsilon_{g_1 n_2} &= \frac{1-\beta_1}{1-\mu\beta_1} \frac{n_1}{n_1+n_2} \varepsilon_{v_1} v_1 && (5.82)
\end{aligned}$$

In this case the optimal policy mix $(\tau, n_2)^*$ necessarily satisfies (5.81-5.82). More explicitly, using (5.27), we derive a sort of MRS:

$$\begin{aligned}
\frac{g_1}{c_1} \frac{\partial u_1/\partial g_1}{\partial u_1/\partial c_1} &= \frac{\frac{\tau}{1-\tau}\frac{\sigma}{1-s}}{1 - \frac{\tau}{1-\tau}\frac{s}{1-s}\frac{s+(1-s)\pi+\sigma-1}{s+(1-s)\pi}} && (5.83)\\
\frac{g_1}{c_1} \frac{\partial u_1/\partial g_1}{\partial u_1/\partial c_1} &= \frac{n_1}{n_1+(1-\rho)\,n_2} \frac{1-\beta_1}{1-\mu\beta_1} \frac{\varepsilon_{v_1}}{\partial u_1/\partial c_1} \frac{v_1}{c_1} - \frac{n_1+n_2}{n_1+(1-\rho)\,n_2} && (5.84)
\end{aligned}$$

In order to ensure the concavity of the welfare function, we require the Hessian matrix to be negative definite.

$$\frac{\partial^2\omega}{\partial\tau^2} < 0 \text{ and } \frac{\partial^2\omega}{\partial\tau^2}\frac{\partial^2\omega}{\partial n_2^2} > \left(\frac{\partial^2\omega}{\partial\tau\partial n_2}\right)^2$$

These inequalities introduce a restriction in the parameter space.

Chapter 6

Concluding remarks and further research

In the current scenario of global economic interdependence, the understanding of the mechanisms at the roots of international flows of financial and human capital should be considered a great priority for economists and policy makers. In this doctoral thesis, we have thus studied the fundamental factors triggering international flows of human and financial capital. This has allowed us to provide insights for policy making.

The thesis is divided in two parts. In the first one, we have focused on international financial capital flows, balance of payments and exchange rate dynamics. In the second one, we have analyzed migration flows to the purpose of discussing immigration policies.

In Chapter 2 we have tracked the accumulation of external debt and the dynamics of the exchange rate in a small open economy. We have focused in particular on the response of the economy to an exogenous demand shock. Our analysis – based on a modified version of the Mundell-Fleming model – has shown that if the real interest rate is greater than the country's growth rate, the shock requires the exchange rate to depreciate in the long run. Indeed, in presence of a Taylor rule, the demand shock triggers a monetary tightening, leading in turn to currency appreciation. In the long run, the exchange rate is eventually more depreciated than its pre-shock value. The implied improvement in competitiveness is reflected in the trade balance. This eventually ensures the economy enough resources to repay the debt. Notice however that this result changes in presence of a negative gap between the real interest rate and the growth rate. In this case, as it has been the case in recent years for several countries, the steady-state level of the exchange rate is more depreciated than the pre-shock value.

We have also checked how introducing imperfect substitutability in international assets affects these results. We have assumed in particular that investors have a bias in favour of domestic assets; therefore, larger debt burdens entail paying higher interest rates. Our results show that in this case, long-run equilibrium more likely requires currency depreciation (relatively to the perfect substitutability case). The reason is that home bias entails higher interest payments and requires thus more resources to finance the debt.

The above results emphasize thus the significance of the impact of interest rate swings on external positions and exchange rate dynamics; this has important implications for policy making. Indeed, were interest rates to reverse back to higher levels, "feasible" debt levels would eventually require significant currency swings. In addition, this effect would be enhanced by home bias (suggested by data) and by changes in investors' expectations.

Chapter 2 has provided a straightforward analysis on external debt dynamics. It carries however the caveats due to the lack of micro-foundations – highlighted in the Introduction. In order to improve on the lack of microfoundation we have built in Chapter 3 a dynamic stochastic general equilibrium model. We also have introduced some features that bulked large in the New Open Macroeconomics literature, such as price rigidities, durable goods, home bias in trade and market imperfections. In particular, given the recent developments in the housing sector and their significant impact on the global economy, we have focused on the role of houses when used as collateral. In presence of debt limits and agents' heterogeneity in their degree of impatience, this can affect consumption patterns through the implied wealth effects.

In our work, we have focused in particular on the linkage between the exchange rate, agents' wealth and international capital flows. We have shown how this mechanism helps propagate internationally all shocks hitting the housing market. Wealth effects – associated to house prices swings – together with income effects – associated to changes in international relative prices – have a significant impact on consumption patterns. This has in turn important implications for the dynamics of the current account.

Our analysis allows extending the seminal result of Becker (1980) and Becker and Foias

(1987, 1994) to an open economy setting. Indeed, if the representative agent of one country is more impatient than the one of the other country, the steady-state external position is defined by the limit imposed by the collateral constraint. This suggests thus that both agents' impatience, on the one hand, and large external imbalances, on the other, can be consistent with well-behaving trajectories toward steady state and sustainable levels of external debt. Moreover, the system is not characterized by unit roots: sufficiently small stochastic shocks do not affect the steady state of the system.

Our analysis of the transition dynamics has studied the response of the economy to i) shocks hitting the housing sector (i.e., a positive preference shock for houses and loosening of the collateral constraint, respectively); ii) technology shocks and iii) the transmission of shocks hitting the Foreign country (i.e., a positive preference shock for tradables and a technology shock, in the Foreign country). We have in particular analyzed the impact of price rigidities and of the monetary policy stance.

Our results are in line with Ferrero et al. (2008) in showing that nominal rigidities do not significantly affect the adjustment dynamics of international real variables, such as the current account. Clearly, they entail larger swings for the exchange rate. Current account dynamics are not significantly affected also by the inclusion/exclusion of house prices in the target of the monetary authority. Our results are thus consistent also with Iacoviello (2005) and Monacelli (2007) but in an open-economy framework: targeting house prices does not improve the adjustment dynamics of (international) real variables. Finally, targeting house prices entails large interest rate and exchange rate fluctuations.

The above results should be interpreted by policy makers with caution. Indeed, while emphasizing well-behaving trajectories toward sustainable steady-state levels of debt, they are based on the hypothesis of sufficiently small shocks (see analysis in Chapter 3). This carries an important word of warning: were greater shocks to take place with dramatic effects on agents expectations, the parallel with the emerging-countries literature à la Krugman (1999) and Aghion et al. (2000) could no longer be easily dismissed.

Chapter 3 has analyzed the transmission channels linking agents' wealth, the housing sector, exchange rate dynamics and the accumulation of external debt. However, while providing a peculiar focus on transmission mechanisms and current account dynamics, some aspects have been left for further research.

Future work should indeed try to use the above mechanisms to come closer to data. While the above research has aimed at isolating effects so as to provide a better understanding of all components, further research should instead combine them so as to proxy reality. Also, introducing capital and adjustment costs for housing investment would certainly add realism and probably allow impulse response functions to closely proxy data.

Notice also that the current scenario of financial crisis and monetary interventions prompts a deeper understanding of optimal monetary policy. In particular, it is important to understand the implications of monetary policy in the context of a world of interdependent open economies, where agents are collateral constrained. Further research should thus focus on optimal monetary policies both for creditor and debtor countries. This could provide interesting hints in light of current trends. By considering an open economy framework, this extension would also add on Monacelli (2007) work. Notice finally that the two-country dimension has also important implications for the response of the economy to monetary shocks. Indeed, as long as the uncovered interest parity condition holds, the monetary policy itself can be subject to significant feed-back effects, as remarked by Canzonieri et al. (2008). Further research should try to disentangle these effects.

In the second part of this thesis we have analyzed human capital flows to study immigration policies. In Chapter 4 we have studied the mechanisms that lead destination countries to implement immigration quotas. Our analysis is based on Benhabib (1996) but incorporates immigrants' heterogeneity in skills and policy implementation costs. We have shown that in absence of enforcement costs, the immigration policy – implemented through a voting system à la Condorcet – is a corner solution both for skilled and unskilled immigrants. In this case, complete closure or opening depend on capital endowments of the median voter; borders are kept open if s/he is rich, closed if poor.

The implementation of immigration quotas entails significant costs and a drain on voters' incomes. Following Magris and Russo (2005), we have thus incorporated these costs and analyzed their impact on the choice of the immigration policy. We have in particular supposed that enforcement costs are a positive function of the tightness of the policy.

Our results show that if immigrants are heterogeneous in skills and the enforcement of quotas is costly, we generally obtain interior solutions. This means that natives choose thus positive quotas both for skilled and unskilled immigrants.

The results in Chapter 4 are the outcome of a comparison between the benefits arising from factor complementarity and the costs associated to a policy that regulates human capital. The arrival of immigrants may however be associated to a drain on government resources; these costs are particularly significant if the government provide a public good to all inhabitants (and thus, also to immigrants) – as it is the case in welfare states. Chapter 5 has thus analyzed the impact of immigration into welfare states, to the purpose of studying optimal immigration (and fiscal) policies. The analysis has assumed a relatively conservative stand so as to contribute to the current debate on immigration policies. Indeed, we have allowed for free-riding in immigrants' contribution to government revenues. Moreover, we have assumed that the government provides a public good that is subject to congestion and maximizes the welfare of natives only; finally, we have supposed that natives are hostile toward newcomers.

Our results show that if natives are not hostile towards foreign workers, migration is always optimal. In presence of hostility, interior solutions are preferable.

We have also characterized the optimal fiscal policy; indeed, both policies are generally interdependent. Notice interestingly that nor hostility nor the evolution of hostility during time has an effect on the optimal fiscal policy. The reason is that while agent's utility is affected by the degree of congestion of the public good, consumers' contribute to government revenues does not depend on it.

Chapter 5 provides an analytical characterization of the mechanisms at the roots of our results – based, when possible, on generic functional forms. Differently from most of the related literature, it also provides a full characterization of the dynamics around the steady state; moreover, it studies the dynamic properties of the steady state.

Our focus on analytical solutions has indeed the benefit of disentangling the roots of outcomes. Thus, by highlighting the source of positive and negative effects on welfare, it provides a solid framework for policy making. Our model has implicitly remarked the importance of factor complementarity and the predominance of economic over cultural factors – when considering the welfare effects of immigration. Complementarity has also important implications when concerning natives' consumption basket. Indeed, if the public and the private good are complement in the consumption bundle, all policies inducing an increase in public-good provision entail a loosening of immigration restrictions.

The analysis of stability properties has also pointed out a possible role of the government in stabilizing the economy: in presence of specific functional forms, the elimination of tax evasion and black-labor markets have indeed stabilization effects.

Our emphasis on analytical closed-form solutions restricts the focus of the analysis to a relatively simple framework. Therefore, this model leaves scope for several possible extensions.

As remarked by the IMF, factor complementary and the implied reallocation of factors may trigger per se immigration flows. Indeed, together with wage differentials, it likely explains part of migration flows. However, weaker forms of complementarity in production would add realism to our analysis and check for robustness of our results. One possible extension could indeed consist in incorporating skills heterogeneity.

Analogously, the formalization of the immigration policy as a rate during time instead of a stock una tantum would come closer to reality. Both changes would however complicate the analytical side of the analysis.

Finally, our bias in favor of an analytical characterization of the dynamics has obliged us to dismiss the discussion on intertemporal inconsistency of optimal policies. We have thus simply assumed that the government commits to a policy, which is optimal in steady state. There are important implications for capital accumulation arising from time inconsistency (see for instance the seminal work of Chamley, 1986). However, in presence of imperfect markets, a better focus on time inconsistency would likely require dynamics to be analyzed with simulation methods[1] and specific functional forms. This has been out of the purpose of the present work but could be subject of further studies.

[1]See also Marcet and Marimon (1998).

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